

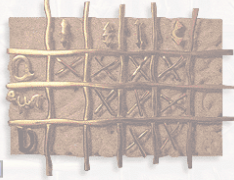
S. Monteil,
LPC – Université Blaise Pascal – in2p3.
[LHCb experiment and CKMfitter group]

Some authoritative literature about the lecture :

- BaBar physics book: <http://www.slac.stanford.edu/pubs/slacreports/slac-r-504.html>
- LHCb performance TDR: <http://cdsweb.cern.ch/record/630827?ln=en>
- A. Höcker and Z. Ligeti: CP Violation and the CKM Matrix. hep-ph/0605217

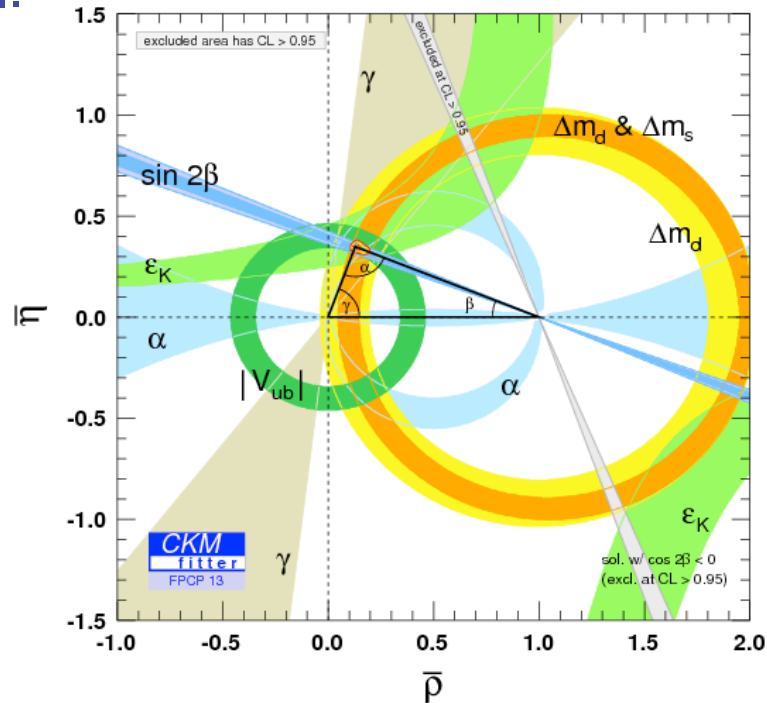
World Averages and Global Fits:

- Heavy Flavour Averaging Group: <http://www.slac.stanford.edu/xorg/hfag/>
- CKMfitter: <http://ckmfitter.in2p3.fr/>
- UTFit: <http://www.utfit.org/>

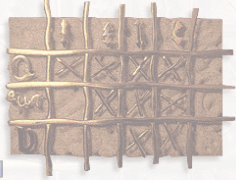


Motivation

- In any HEP physics conference summary talk, you will find this plot, stating that (heavy) flavours and CP violation physics is a pillar of the Standard Model.

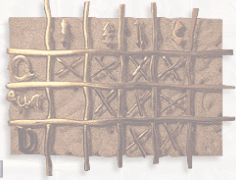


- One objective of these series of lectures is to undress this plot.



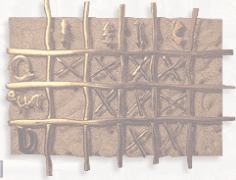
Disclaimers

- This is an experimentalist point of view on a subject which is all about intrications between experiment and theory.
- The main machines in question here are the TeVatron (Fermilab, US), PEP-II (SLAC, US), KEKB (KEK, Japan) and LHC (CERN, EU). Former experiments played a pioneering role: LEP (CERN, EU) and CLEO (CESR, US).
- Most of the material concerning global tests of the SM and above is taken from the CKMfitter group results (assumed bias) and Heavy Flavour Averaging Group (and hence the experiments themselves). I borrowed materials in presentations from colleagues which I tried to cite correctly.



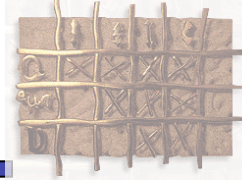
The outline by Chapters

1. History and recent past of the parity violation experiments.
The discovery of the CP violation.
2. Observables and measurements relevant to study CP violation.
3. The global fit of the SM.
4. New Physics exploration with current data: two examples.



2.0 CP-conserving Observables

- Magnitudes of the matrix elements:
 - $|V_{ud}|$ and $|V_{us}|$.
 - $|V_{ub}|$ and $|V_{cb}|$.
- Frequency of B^0 oscillations:
 - Δm_d and Δm_s .
- These are all CP-conserving quantities. But they do bring information on CP violation.



2.0 The matrix elements $|V_{ud}|$ and $|V_{us}|$.

- These matrix elements do not exhibit dependencies to the KM weak phase.

- They however play a role in the global fit through the determination of the two others parameters λ and A .

The magnitudes: $|V_{us}|$

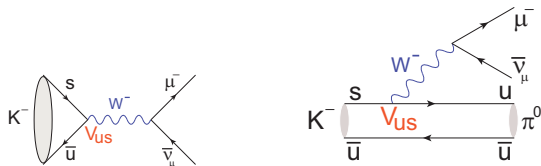
$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

2008 RPP

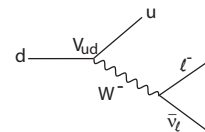
$$|V_{us}| = 0.2255 \pm 0.0019$$

$|V_{us}|$ from kaon decays

□ New averages from FlaviaNet Kaon Working Group, arXiv:1005.2323 [hep-ph] (2010), see also KLOE (Archilli, 1085)



- $K_{\ell 3}$: $|V_{us}|f_+(0) = 0.2163(5)$ or $|V_{us}| = 0.2254 \pm 0.0013$ with $f_+(0) = 0.959(5)$ (lattice, Boyle et al., arXiv1004:0886 (2010))
- $K_{\ell 2}$: $\frac{|V_{us}|f_K}{|V_{ud}|f_\pi} = 0.2758(5)$ or $\frac{|V_{us}|}{|V_{ud}|} = 0.2312 \pm 0.0013$ with $f_K/f_\pi = 1.193(6)$ (lattice average)
- Combining, obtain $|V_{us}|(K) = 0.02253 \pm 0.0009$



2008 RPP

The magnitudes: $|V_{ud}|$

$$|V_{ud}| = 0.97418 \pm 0.00027$$

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Best determinations in superallowed $0^+ \rightarrow 0^+$ nuclear β decays.

Recent analysis from Hardy and Towner PRC **79** (2009) 055502 yields:

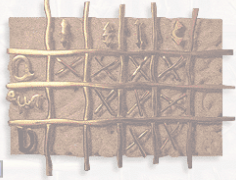
$$|V_{ud}| = 0.97425 \pm 0.00022$$

Porter in ICHEP' 10

The determination of these magnitudes are both limited by the knowledge of the hadronic parameters (strong interaction modelling). State of the art as in PDG2012:

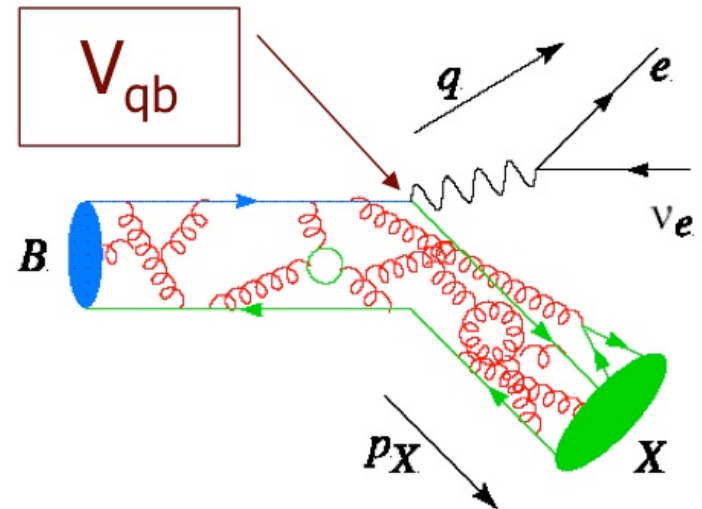
$$|V_{ud}| = 0.97425 \pm 0.00022$$

$$|V_{us}| = 0.2252 \pm 0.0009.$$

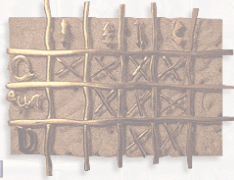


2.1 The Semileptonic branching ratios: the $|V_{ub}|$ and $|V_{cb}|$ matrix elements.

- The magnitude $|V_{ub}|$ is key observable in constraining the CKM profile. It basically determines the side R_u of the CKM triangle.
- Pioneered in CLEO and LEP experiments. Nowadays B factories results dominate by far.
- The matrix element V_{cb} enters everywhere in the triangle: as a normalization and in dependencies in some observables.
- There are two ways to access the matrix elements: the inclusive (whatever the charmless X is) and exclusive (specific decays – mainly $[B \rightarrow \pi | \nu]$) decay rates.

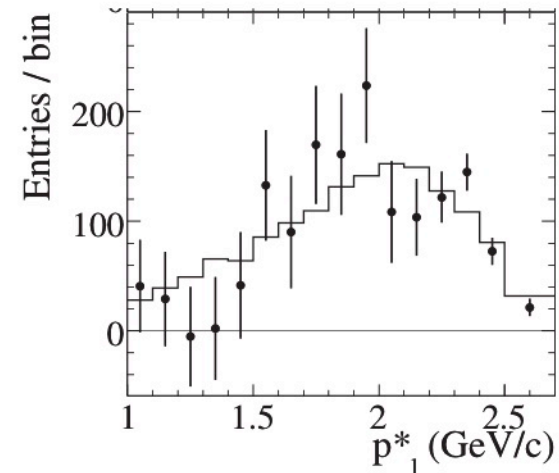


$$R_u = \left| \frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} \right| = \sqrt{\bar{\rho}^2 + \bar{\eta}^2},$$

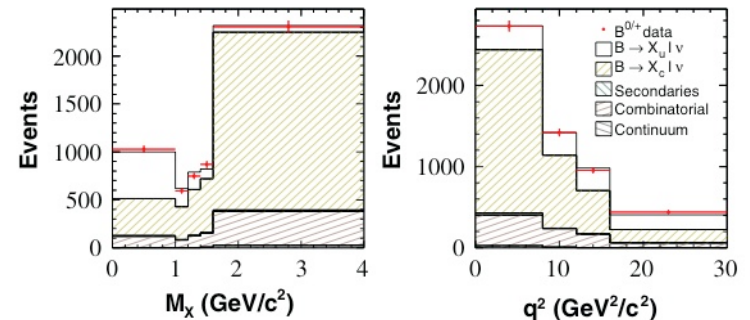


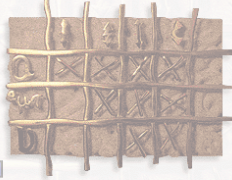
2.1 The Semileptonic branching ratios: the $|V_{ub}|$ and $|V_{cb}|$ matrix elements.

- The experimental technique shared by most of the analyses is to fully reconstruct one B decay (hadronic mode) and look at the other B of the event.
- Though the branching fractions of the fully reconstructed mode is small, the full kinematics of the other decay is constrained.
- Apply to exclusive and **inclusive modes**.
- It's in general crucial (at least for V_{ub}) that the background is measured in off-peak data.



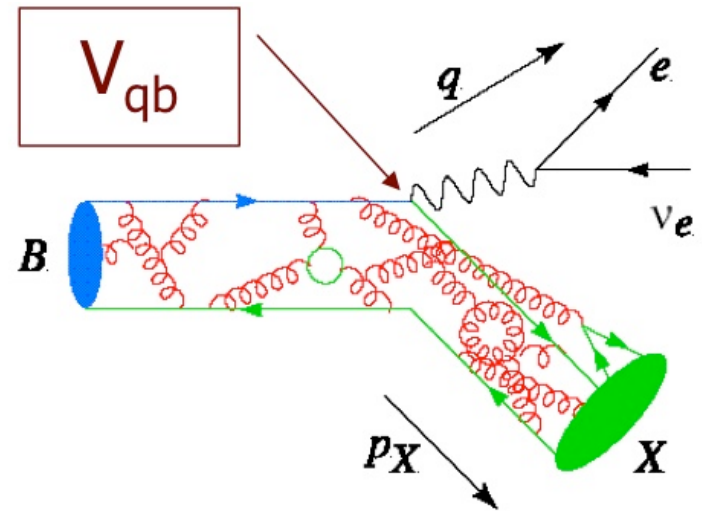
Top: Babar p_1 bkgd subtracted
Bottom: Belle M_X and q^2





2.1 The Semileptonic branching ratios: the $|V_{ub}|$ and $|V_{cb}|$ matrix elements.

- The measurement of the branching fraction relies on the lepton detection and identification above a given threshold.
- To relate the measurements to the theoretical prediction, one has to extrapolate the experimental spectra and hence rely on models.
- As suggested by the diagram on the right, the hadronic content of the decay is rich. Several theoretical techniques (see Descotes in GIF10).
- From the experimental point of view, charmless semileptonic are very difficult to separate from charm (in a ratio 1/150) .
- Very intense activity in this field of theory/ experiment collaboration.

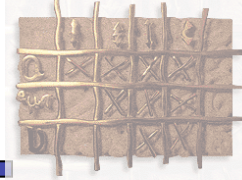


The decay rate from the parton level:

$$\Gamma_0 \equiv \Gamma(b \rightarrow c[u]\ell\bar{\nu}) = \frac{G_F^2 |V_{c[u]b}|^2}{192\pi^3} m_b^5$$

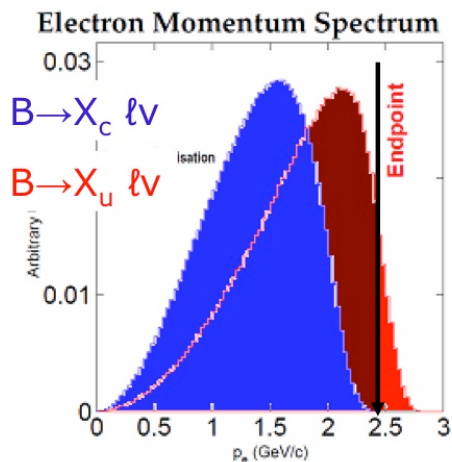
to

$$\frac{\partial^3 \Gamma}{\partial E_\ell \partial q^2 \partial m_X} = \underbrace{\Gamma_0}_{\text{free quark decay}} \times f(E_\ell, q^2, m_X) \times \underbrace{\left(1 + \sum_n C_n \left(\frac{\Lambda_{QCD}}{m_b} \right)^n \right)}_{\text{Perturbative + non-perturbative corrections}}$$



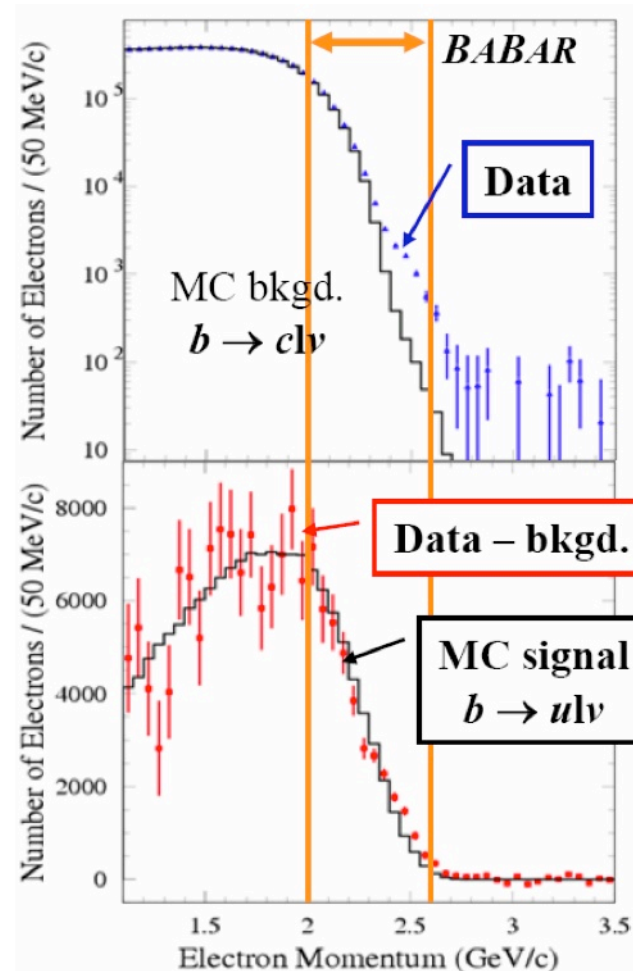
2.1 The Semileptonic branching ratios: the $|V_{ub}|$ and $|V_{cb}|$ matrix elements.

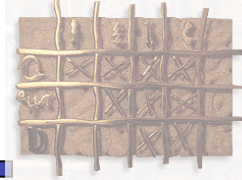
- Inclusive $|V_{ub}|$ measurements: lepton endpoint.
- It's tempting to consider the pure $b \rightarrow u$ region.
- But for higher signal efficiency, the theoretical error is smaller. Get a compromise. Typically the cut is defined larger than 2 GeV.



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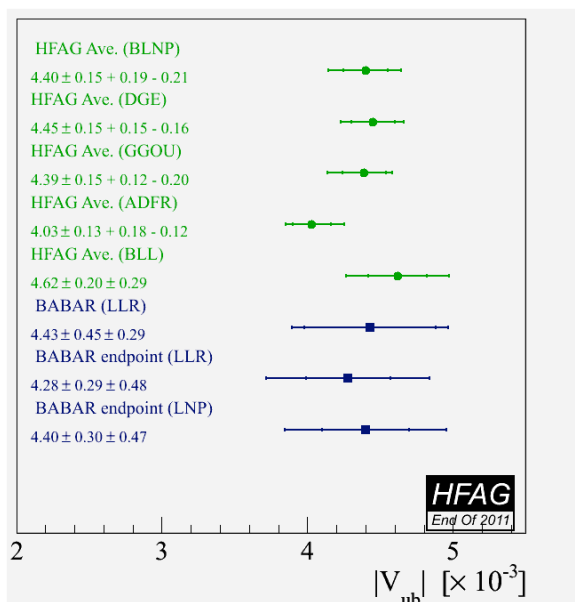
Lyon 2013





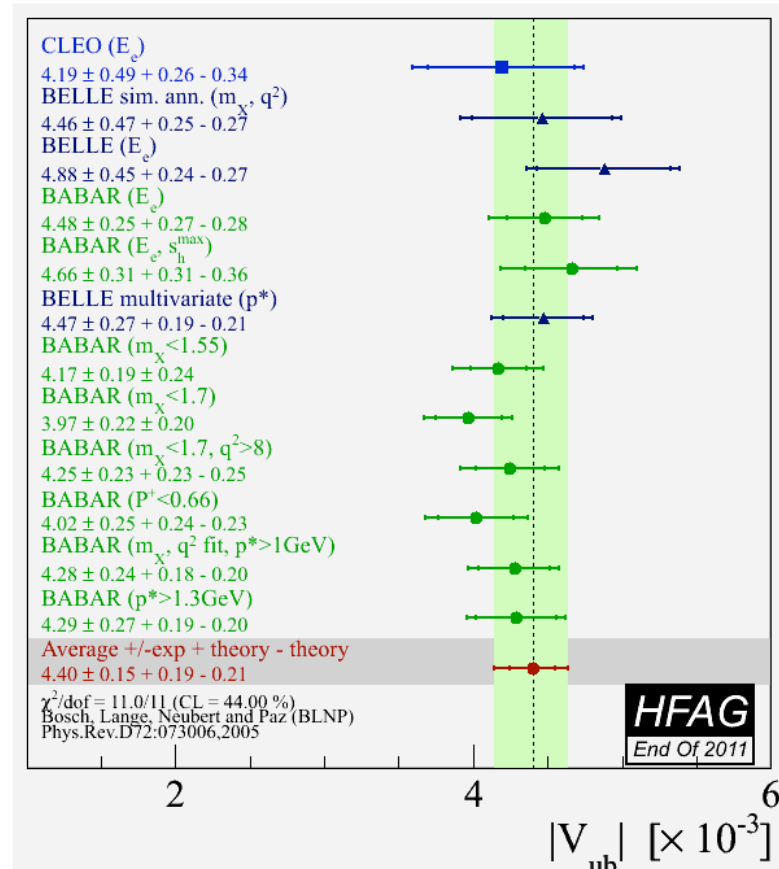
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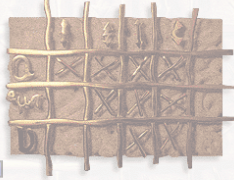
- Summary of inclusive $|V_{ub}|$ determinations:
- Shown are the extrapolation within the BLNP scheme. BLNP PRD 72 (2005), DGE arXiv:0806.4524, GGOU JHEP 0710 (2007) 058, ADFR Eur Phys J C 59 (2009) 831



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2.1 The Semileptonic branching ratios: the $|V_{ub}|$ and $|V_{cb}|$ matrix elements.

- Summary of exclusive $|V_{ub}|$ determinations:

$$\frac{d\Gamma(B \rightarrow \pi \ell \nu)}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{24\pi^3} f_+(q^2)^2 p_\pi^3$$

form factors

phase space

$$\frac{d\Gamma(B \rightarrow D \ell \nu)}{dw} = \frac{G_F^2 |V_{cb}|^2}{48\pi^3} \mathcal{G}(w)^2 \Phi(w)$$

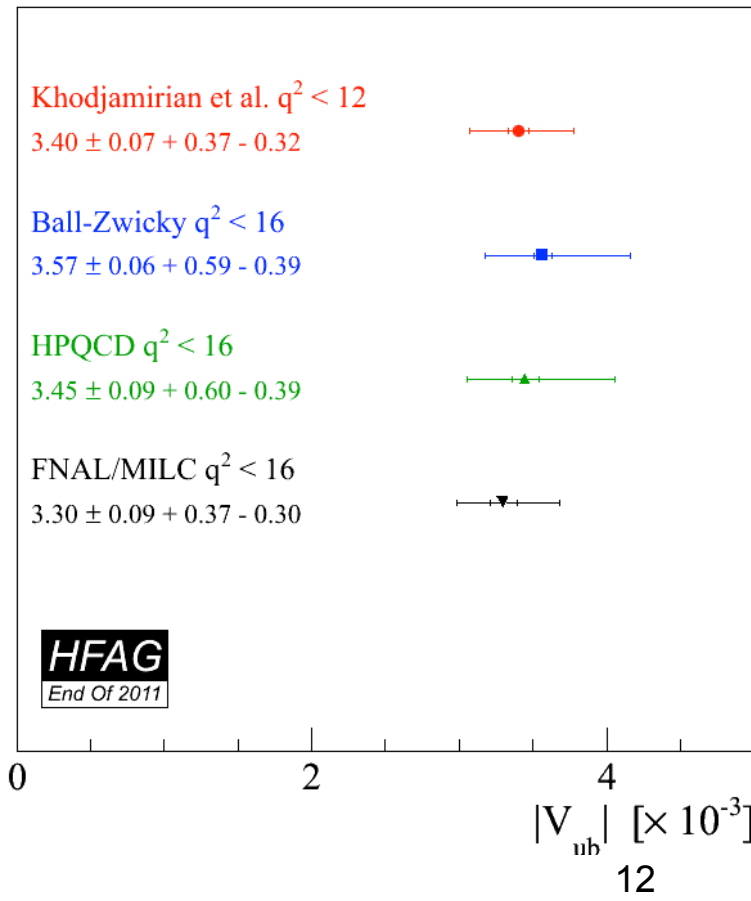
Parameterization of FF improved by mapping to variable with limited range and use of unitarity and analyticity

$w \equiv \frac{m_B^2 + m_{D^*}^2 - q^2}{2m_B m_{D^*}}; 1 < w < 1.504$

$\mathcal{G}(w) = \mathcal{G}(1) [1 - 8\rho^2 z + (51\rho^2 - 10)z^2 - (252\rho^2 - 84)z^3]$

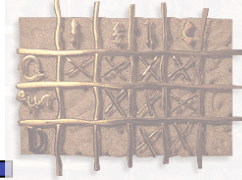
$z = \frac{\sqrt{w+1} - \sqrt{2}}{\sqrt{w+1} + \sqrt{2}}$

D^* boost in the B rest frame



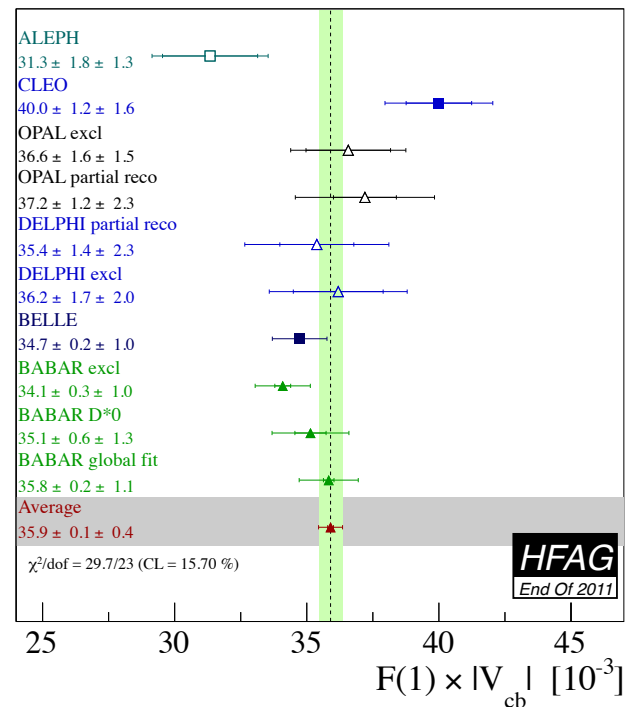
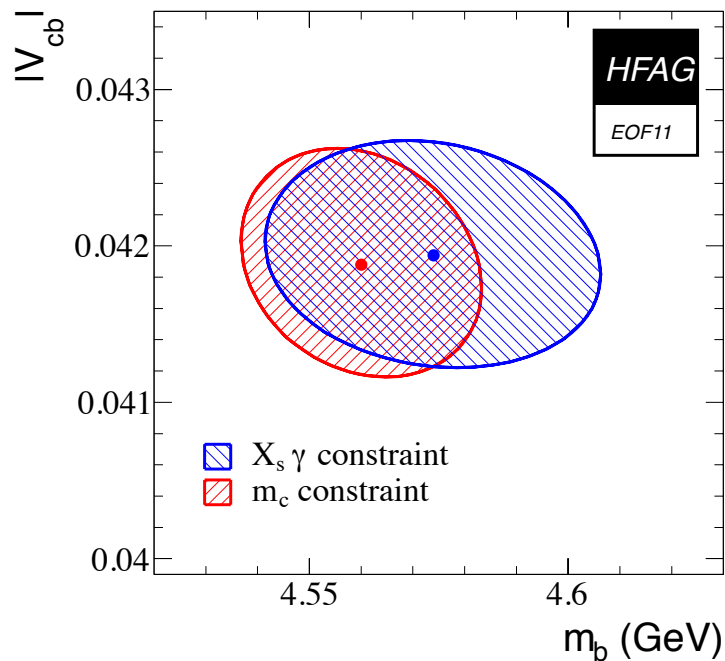
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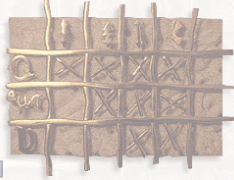
- FF can be as well calculated on the lattice. Significant progresses made recently.



2.1 The Semileptonic branching ratios: the $|V_{ub}|$ and $|V_{cb}|$ matrix elements.

- Summary of inclusive and exclusive $|V_{cb}|$ determinations:

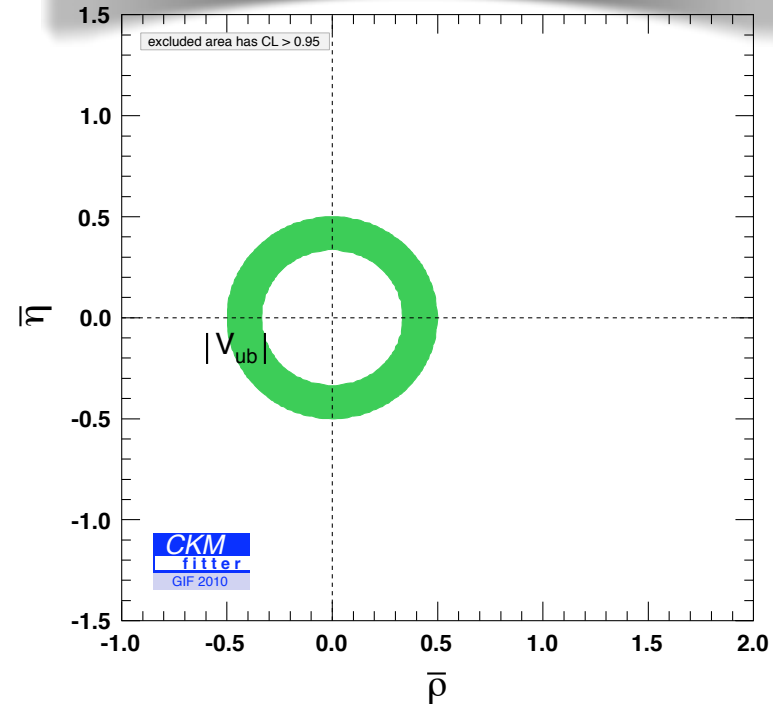




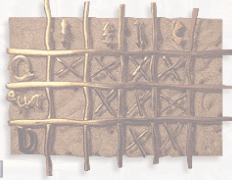
2.1 The Semileptonic branching ratios: the $|V_{ub}|$ and $|V_{cb}|$ matrix elements.

- A typical theoretical uncertainty of 10% on the $|V_{ub}|$ measurement/extraction.
- Inclusive and exclusive determinations of $|V_{ub}|$ marginally agree (the trend is the same for $|V_{cb}|$).
- Intense theoretical effort undergoing in the field.
- Dramatic progresses from LQCD can be expected.

$$|V_{ub}| = (4.15 \pm 0.49) \times 10^{-3}$$



$$|V_{cb}| = (40.9 \pm 1.1) \times 10^{-3}$$

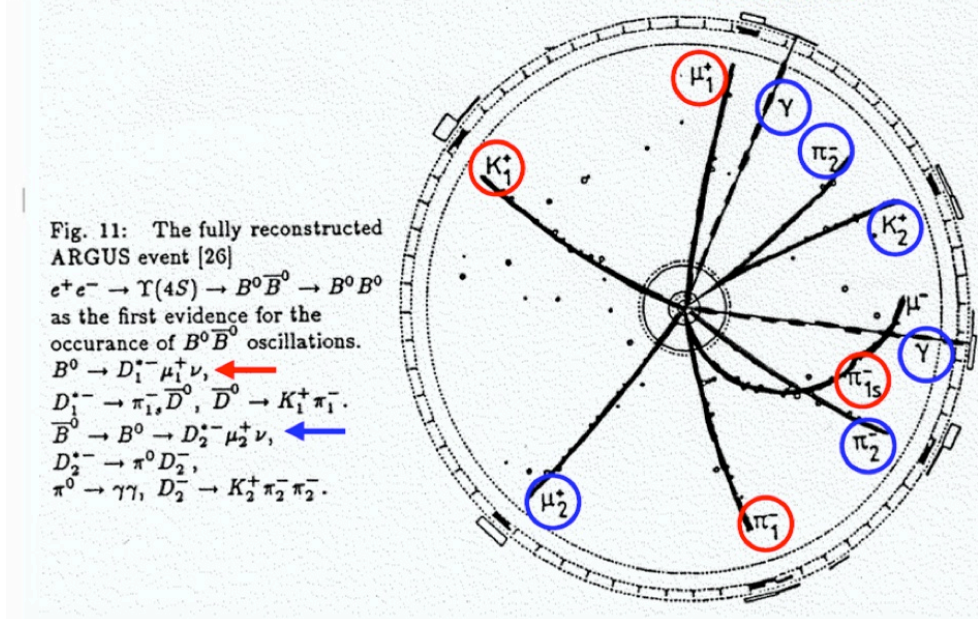


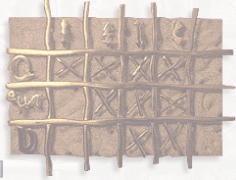
2.2 The oscillation frequencies: Δm_d and Δm_s .

- As we have seen in the kaon system, weakly decaying neutral mesons can mix.
- The B^0 mixing first observation was in 1987 by the Argus collaboration:

B^0 -mixing: First Observation at Argus, DESY, 1987

PLB192, 245 (1987)





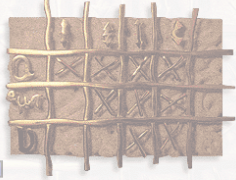
2.2 The oscillation frequencies: Δm_d and Δm_s .

- In the case of weakly decaying neutral mesons (K^0 , D^0 , B^0 , B_s), the mass eigenstates (which propagate) are a superposition of the flavour states. The example of the B^0 in presence of CP violation:

$$\begin{aligned} |B_L\rangle &= \frac{1}{\sqrt{2}}(p|B^0\rangle + q|\bar{B}^0\rangle) \\ |B_H\rangle &= \frac{1}{\sqrt{2}}(p|B^0\rangle - q|\bar{B}^0\rangle) \end{aligned}$$

- The time evolution of these mass states is derived by solving the Schrödinger equation for the hamiltonian $H = M - i\Gamma/2$:

$$|B_{L,H}\rangle = e^{-i(M_{L,H} - i\frac{\Gamma_{L,H}}{2})t} \cdot |B_{L,H}(t=0)\rangle$$



2.2 The oscillation frequencies: Δm_d and Δm_s .

- Intermediate calculation and definitions:

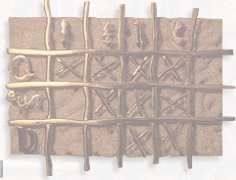
$$|B^0(t)\rangle = (g_+(t)|B^0\rangle + \frac{q}{p}g_-(t)|\bar{B}^0\rangle)$$

$$|\bar{B}^0(t)\rangle = (\frac{p}{q}g_-(t)|B^0\rangle + g_+(t)|\bar{B}^0\rangle)$$

$$g_+(t) = e^{-i(m_B - i\frac{\Gamma_B}{2})t} \left[\cosh\frac{\Delta\Gamma_B t}{4} \cos\frac{\Delta m_B t}{2} - i \sinh\frac{\Delta\Gamma_B t}{4} \sin\frac{\Delta m_B t}{2} \right],$$

$$g_-(t) = e^{-i(m_B - i\frac{\Gamma_B}{2})t} \left[-\sinh\frac{\Delta\Gamma_B t}{4} \cos\frac{\Delta m_B t}{2} + i \cosh\frac{\Delta\Gamma_B t}{4} \sin\frac{\Delta m_B t}{2} \right]$$

- We defined here the mass difference $\Delta m_d = M_H - M_L$ and width (lifetime) difference $\Delta\Gamma_d = \Gamma_H - \Gamma_L$. Δm_B governs the speed of the oscillations.

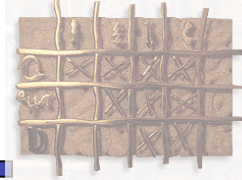


2.2 The oscillation frequencies: Δm_d and Δm_s .

- The master formulae to get a B^0 produced at $t=0$ decaying in a final state f (neglecting $\Delta\Gamma$ in case of the B^0):

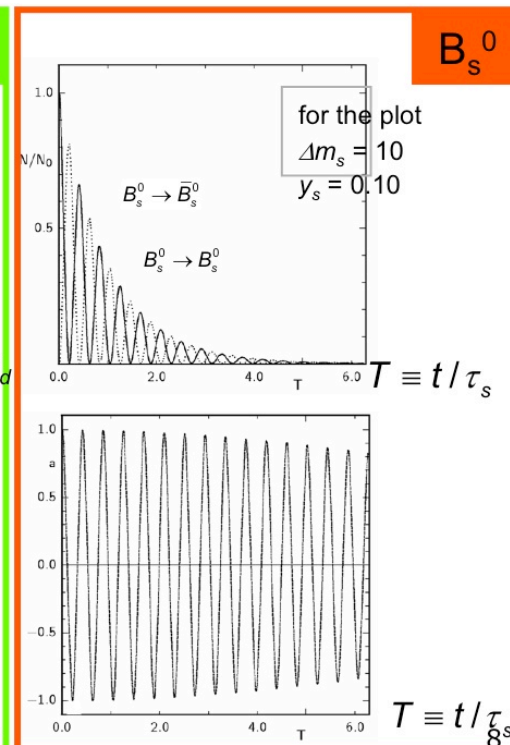
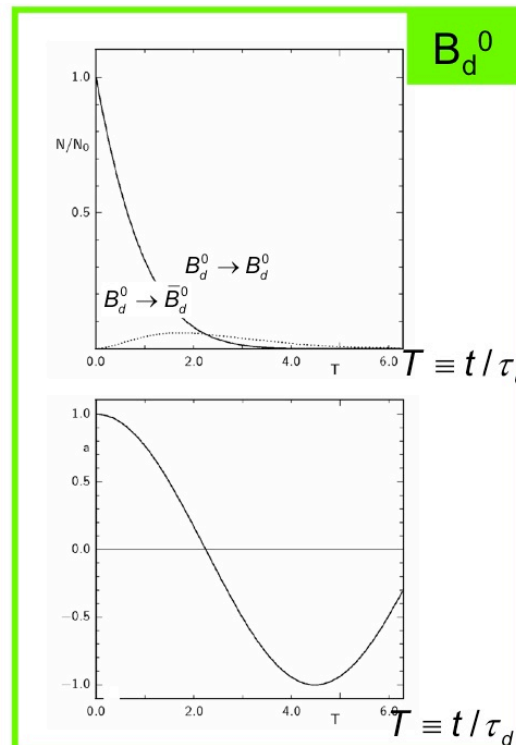
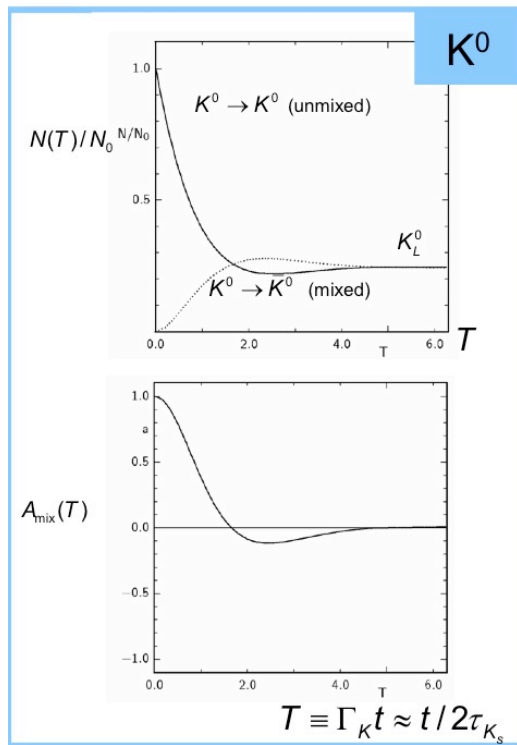
$$\begin{aligned}
 P(B^0(0) \rightarrow f) &= \frac{e^{-\Gamma\tau}}{2} [(1 + \cos \Delta mt) |\langle f | H | B^0 \rangle|^2 \\
 &+ (1 - \cos \Delta mt) \left| \frac{q}{p} \right|^2 |\langle f | H | \bar{B}^0 \rangle|^2 \\
 &- 2 \sin \Delta mt \cdot \text{Im} \left(\left| \frac{q}{p} \right| |\langle f | H | B^0 \rangle| \cdot |\langle f | H | \bar{B}^0 \rangle|^* \right)].
 \end{aligned}$$

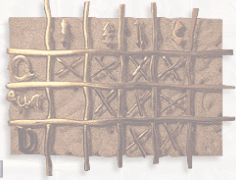
$$\begin{aligned}
 P(\bar{B}^0(0) \rightarrow f) &= \frac{e^{-\Gamma\tau}}{2} [(1 + \cos \Delta mt) |\langle f | H | \bar{B}^0 \rangle|^2 \\
 &+ (1 - \cos \Delta mt) \left| \frac{p}{q} \right|^2 |\langle f | H | B^0 \rangle|^2 \\
 &- 2 \sin \Delta mt \cdot \text{Im} \left(\left| \frac{p}{q} \right| |\langle f | H | B^0 \rangle| \cdot |\langle f | H | \bar{B}^0 \rangle|^* \right)].
 \end{aligned}$$



2.2 The oscillation frequencies: Δm_d and Δm_s .

- Time evolution in plots:





2.2 The oscillation frequencies: Δm_d and Δm_s .

- Which observable in order to look at the time evolution ?
- If we only consider the B^0 mixing in absence of CP violation (remember that it is so tiny in the B mixing that it has not been observed yet):

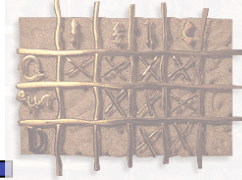
$$|\langle B^0 | H | \bar{B}^0(t) \rangle|^2 = \frac{e^{-\Gamma t}}{2} (1 + \cos \Delta m t)$$

$$|\langle \bar{B}^0 | H | B^0(t) \rangle|^2 = \frac{e^{-\Gamma t}}{2} (1 - \cos \Delta m t)$$

- We want to compare the number of mixed and unmixed events along the evolution. Define the time dependent asymmetry:

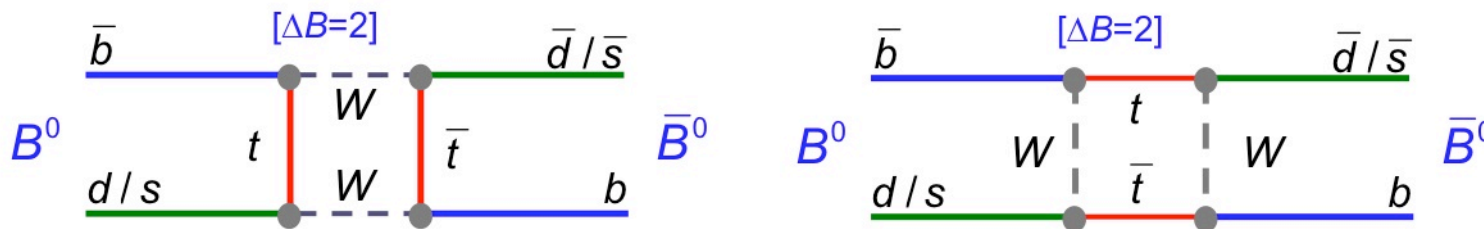
$$A_{\text{mix}} = \frac{N(B^0 \rightarrow B^0) - N(B^0 \rightarrow \bar{B}^0)}{N(B^0 \rightarrow B^0) + N(B^0 \rightarrow \bar{B}^0)} = \cos(\Delta m_d t)$$

- Note: in the case of B_s , the width difference is no more negligible. Complete treatment in LHCb TDR.



2.2 The oscillation frequencies: Δm_d and Δm_s .

- In the Standard Model the short distance contribution is given by the following diagrams dominated in the loop by the top quark contribution.



- and Δm_d is given by:

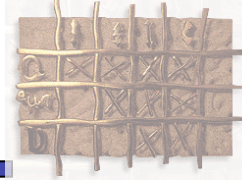
$$\Delta m_d = \frac{G_F^2}{6\pi^2} \eta_B m_{B_d} f_{B_d}^2 B_d m_W^2 S(x_t) |V_{td} V_{tb}^*|^2$$

Non pert. QCD correction.
Main uncertainty.

The weak part we
are searching for.

Pert. QCD correction to Inami-Lim function.

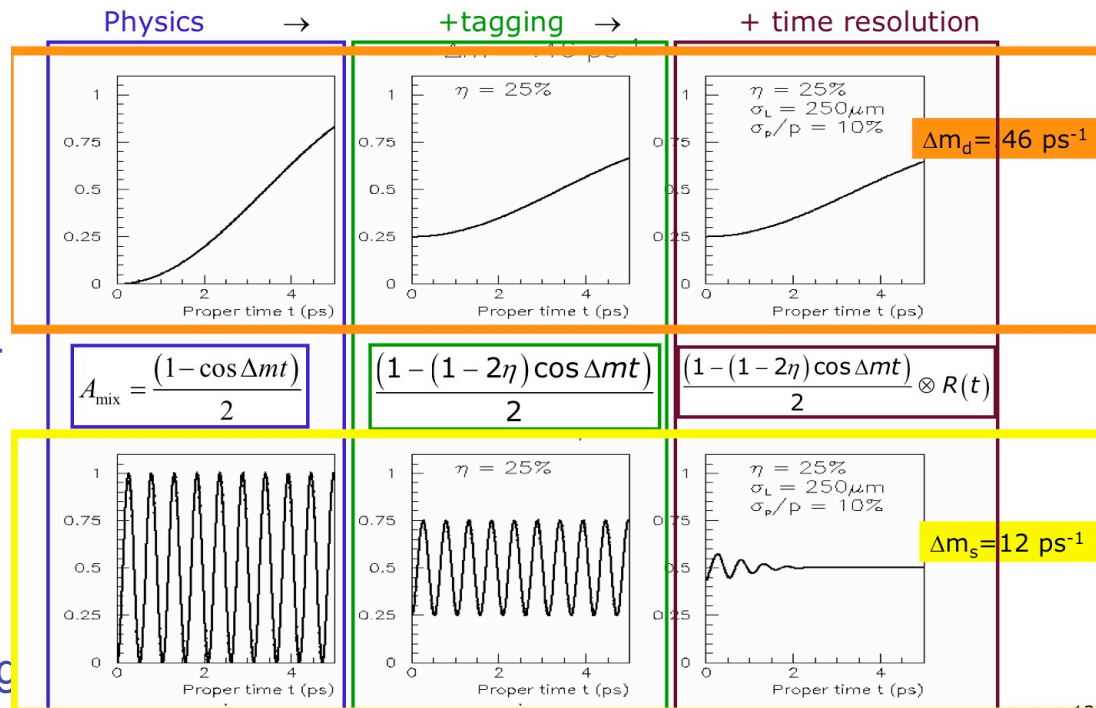
Inami-Lim function describing the content of the box. Top quark dominating.



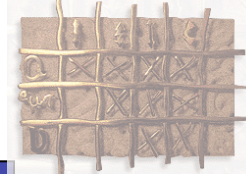
2.2 The oscillation frequencies: Δm_d and Δm_s .

The measurements requires several ingredients:

- Reconstruct the flavour at the decay time
Either use a fully flavour specific hadronic mode or a tag the charge with direct semileptonic decays.
- Reconstruct the decay time. Requires excellent vertexing capabilities (in particular to reconstruct the fast B_s oscillations).
- Reconstruct the flavour at production time (see Jacques's lecture). This is the key ingredient. Made easiest at the B factories where the B mesons are coherently evolving. The flavour of one B at its decay time gives the flavour of the companion at the same time.



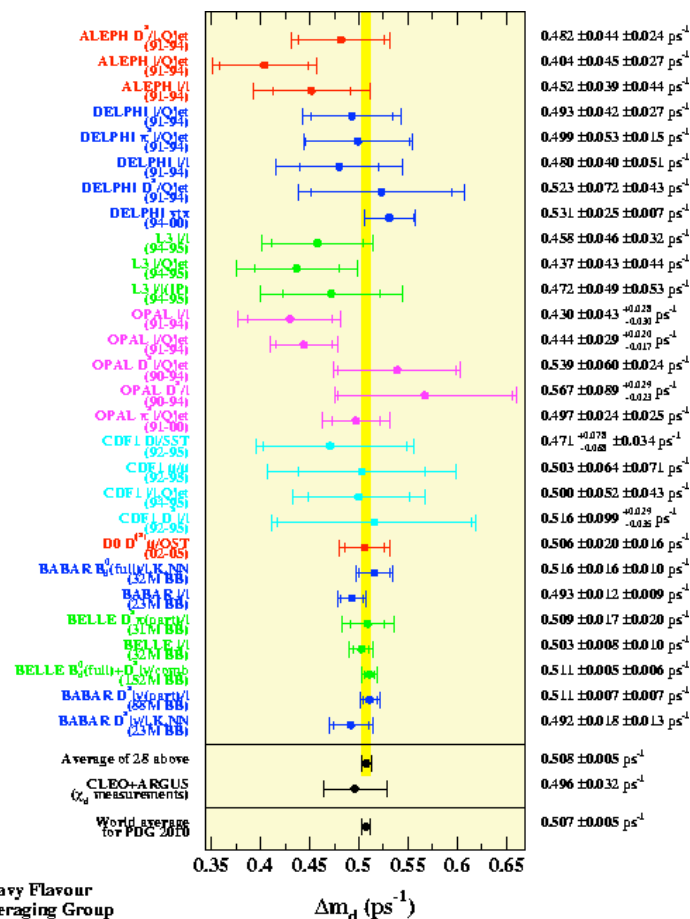
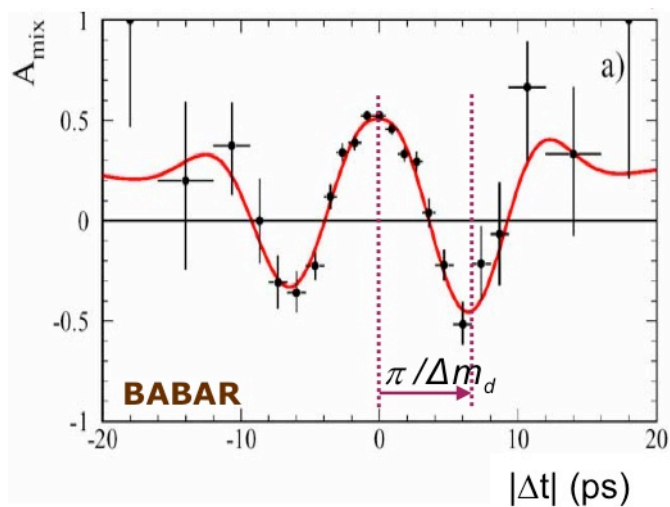
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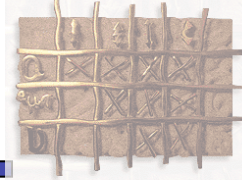


2.2 The oscillation frequencies: Δm_d

Results for the oscillation frequency measurements:

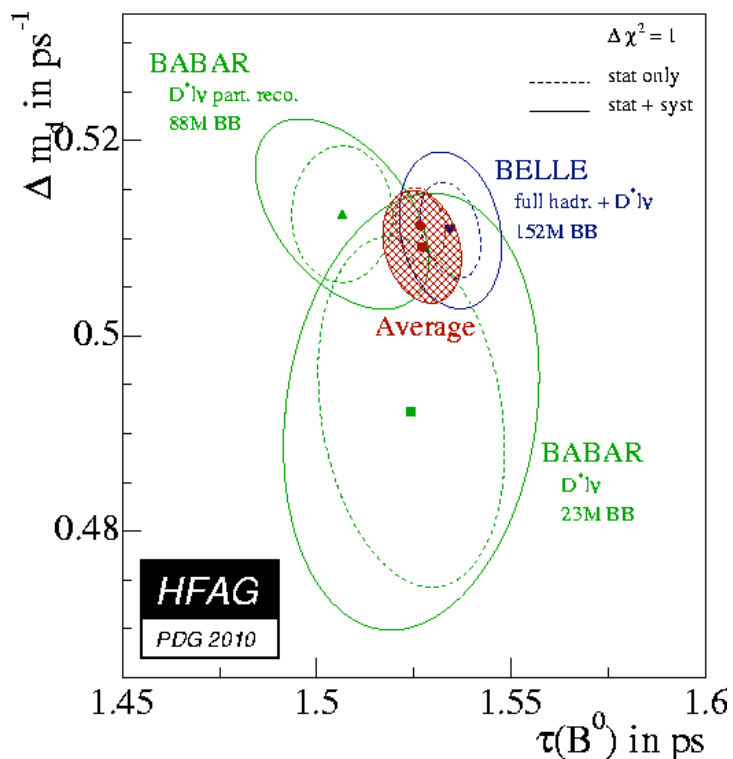
- The BaBar example: this is a fantastic measurement among thirty !



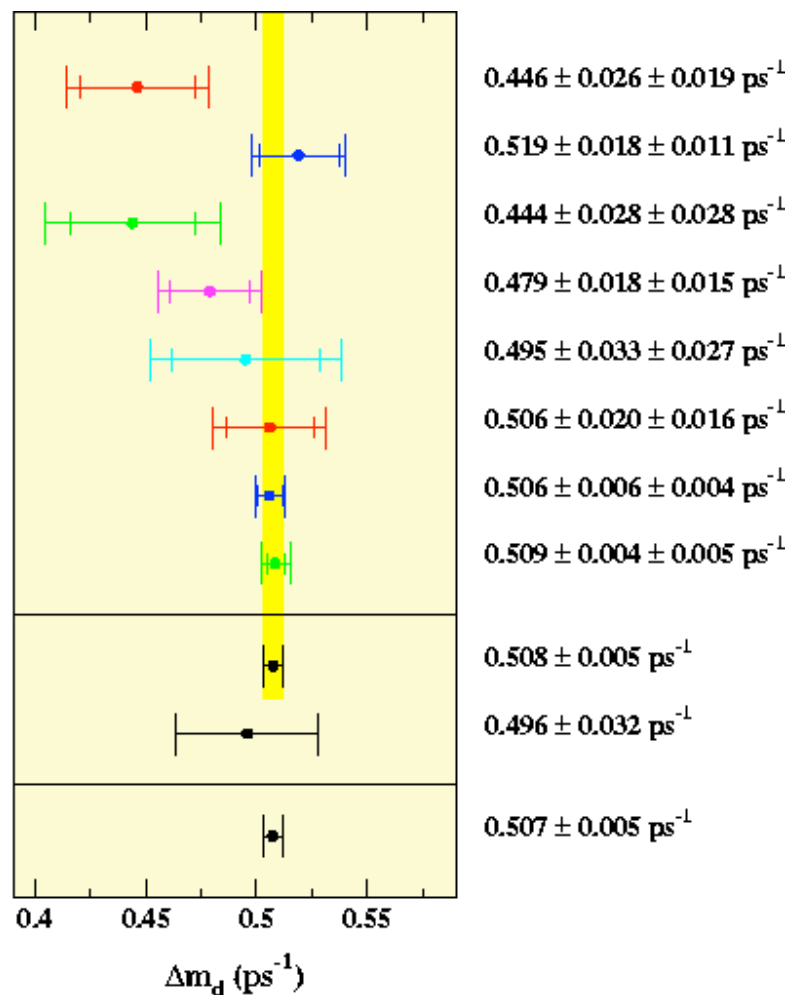


2.2 The oscillation frequencies: Δm_d

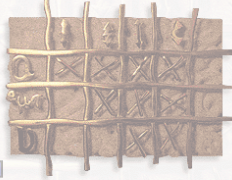
Sorting the results by experiments, it becomes obvious that B factories are dominating the WA.



- ALEPH (3 analyses)
- DELPHI (5 analyses)
- L3 (3 analyses)
- OPAL (5 analyses)
- CDF1 (4 analyses)
- D0 (1 analysis)
- BABAR (4 analyses)
- BELLE (3 analyses)
- Average of above after adjustments
- CLEO+ARGUS (χ_d measurements)
- World average for PDG 2010



* HFLAG average without adjustments
Lyon 2013



2.2 The oscillation frequencies: Δm_d and Δm_s .

$$\Delta m_d = \frac{G_F^2}{6\pi^2} \eta_B m_{B_d} f_{B_d}^2 B_d m_W^2 S(x_t) |V_{td} V_{tb}^*|^2$$

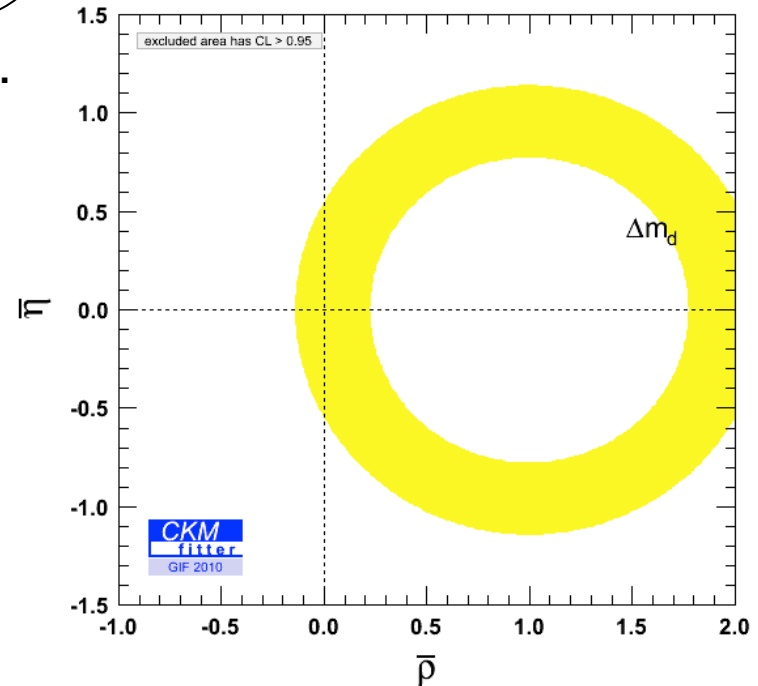
Non pert. QCD correction. Main uncertainty.
The weak part we are searching for.

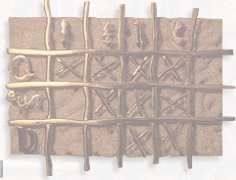
Pert. QCD correction to Inami-Lim function.

Inami-Lim function.

$$R_t = \left| \frac{V_{td} V_{tb}^*}{V_{cd} V_{cb}^*} \right| = \sqrt{(1 - \bar{\rho})^2 + \bar{\eta}^2}$$

- The constraint on the Wolfenstein parameters is entirely dominated by the calculation on the Lattice of the product decayconstant*bagfactor.
- There is a way out to improve the precision with the Bs mixing measurement.





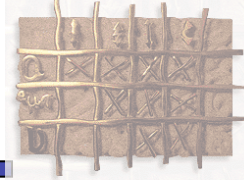
2.2 The oscillation frequencies: Δm_d and Δm_s .

- Though Δm_s only depends marginally on the Wolfenstein parameters, it helps a lot in reducing the LQCD uncertainty. Actually, the ratio:

$$\xi = \frac{f_{B_s} \sqrt{B_s}}{f_{B_d} \sqrt{B_d}}$$

is much better determined (better than 5 %) than each of its argument. Δm_s is improving the knowledge we have on the B_d product *decayconstant X bagfactor*.

- Note: in the global CKM fit, we don't use anymore the *zeta* parameter but directly the ratios of decay constants and bag factors per species.



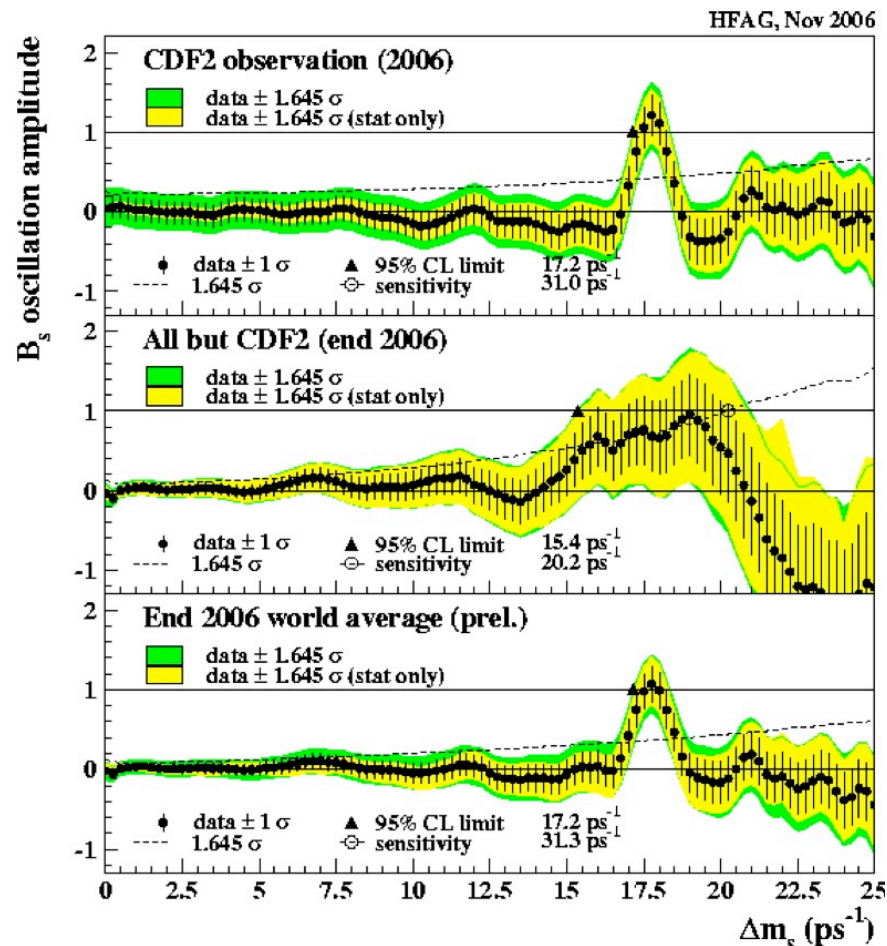
2.2 The oscillation frequencies: Δm_s .

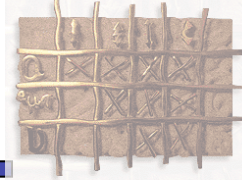
- The CDF experiment managed to resolve the fast oscillations of the B_s and measured the oscillation frequency Δm_s in 2006 with a remarkable accuracy. It was the end of a long search starting at LEP in the early nineties.

- Amplitude method for combining limits:

$$P(B_s^0 \rightarrow \bar{B}_s^0) = \frac{e^{-t/\tau}}{2} \cdot (1 + \mathcal{A} \cos(\Delta m_s t))$$

- \mathcal{A} is measured at each Δm_s hypothesis.
- $\mathcal{A}=0$: no oscillation is seen.
- $\mathcal{A}=1$: oscillation are observed.

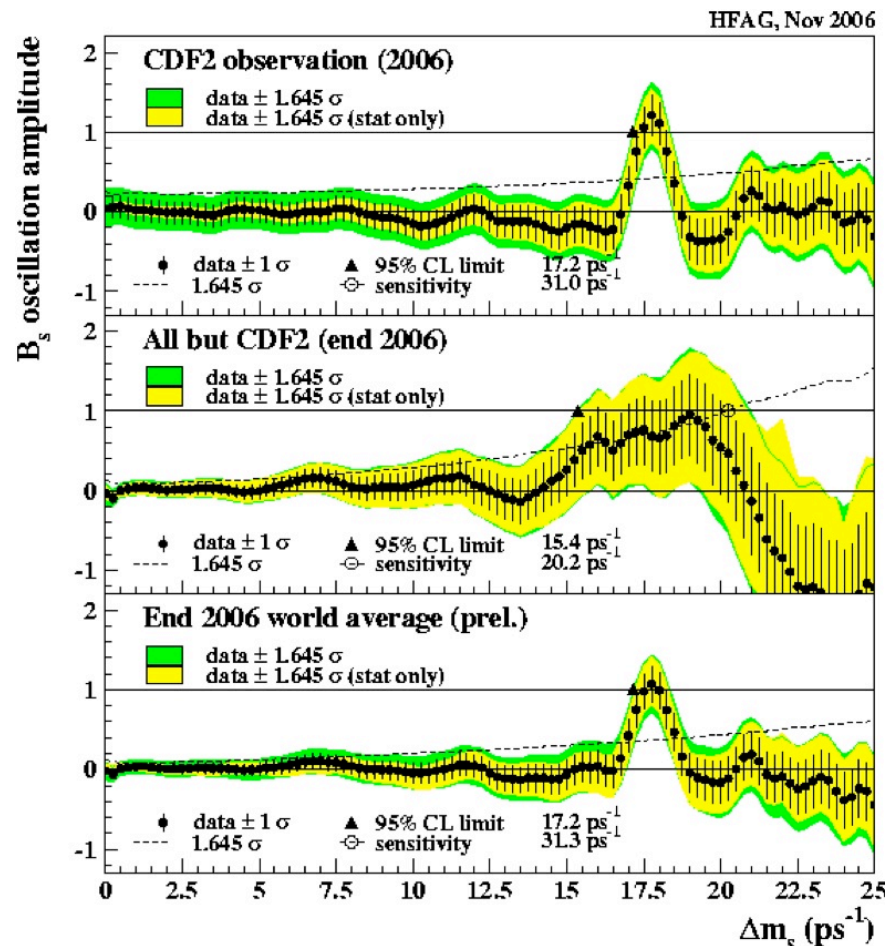


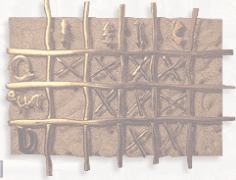


2.2 The oscillation frequencies: Δm_s .

- Digression: looking at the intermediate plot (all experiments but CDF), one sees a structure of the amplitude, yielding to set a limit very close to the CDF measurement.

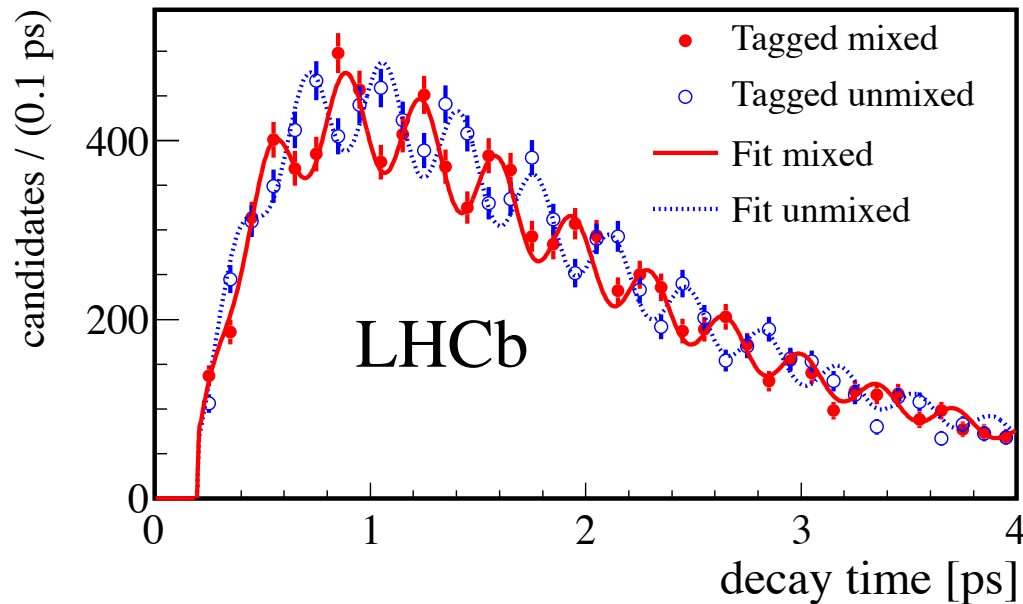
- This was basically driven by the LEP experiments constraints, very close eventually to resolve the B_s oscillations. It was a 2σ effect which was confirmed ... That happens also [Never take 2σ effects too seriously !]





2.2 The oscillation frequencies: Δm_s nowadays

- decay time resolution is of utmost importance for such measurements. A textbook illustration of the LHCb performance can be found in the frequency of the B_s mixing, resolved here in the decay $B_s \rightarrow D_s \pi$:(details in Yasmine's seminar).

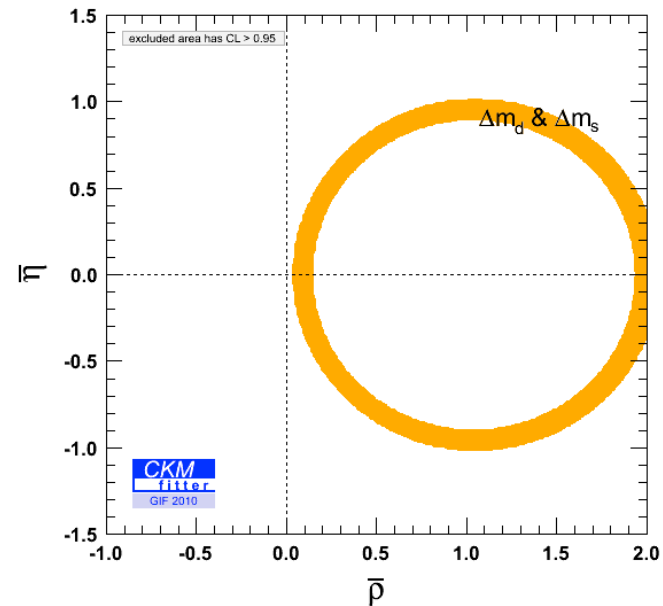
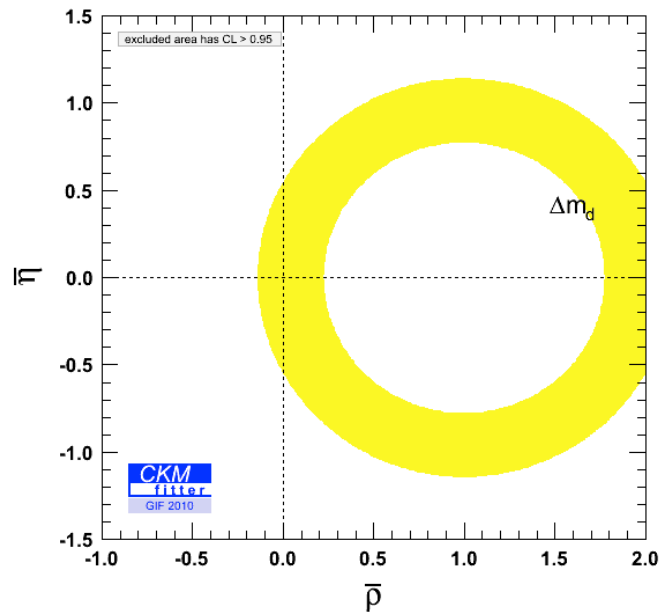


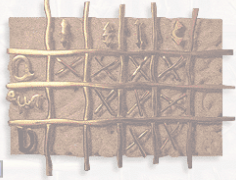
New J. Phys. 15 (2013) 053021 (1 fb^{-1})



2.2 The oscillation frequencies: Δm_d and Δm_s .

- The simultaneous fit of the two oscillation frequencies yield a dramatic improvement in the constraint.

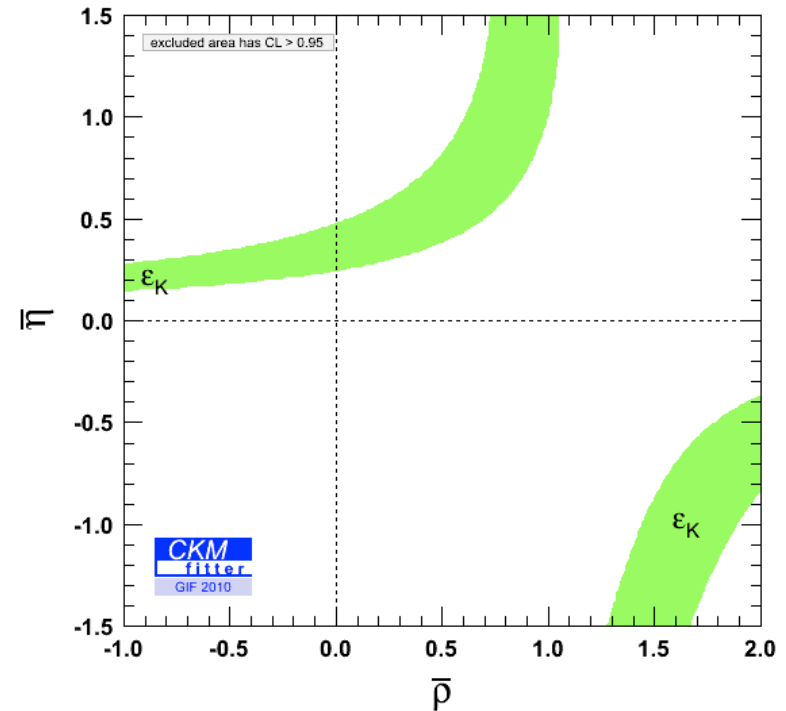


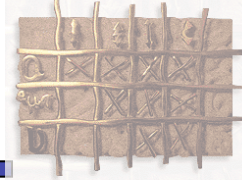


2.3 The neutral kaon mixing $|\varepsilon_K|$.

$$|\varepsilon_K| = \frac{G_F^2 m_W^2 m_K f_K^2}{12\sqrt{2}\pi^2 \Delta m_K} B_K \left(\eta_{cc} S(x_c, x_c) \text{Im}[(V_{cs} V_{cd}^*)^2] + \eta_{tt} S(x_t, x_t) \text{Im}[(V_{ts} V_{td}^*)^2] + 2\eta_{ct} S(x_c, x_t) \text{Im}[V_{cs} V_{cd}^* V_{ts} V_{td}^*] \right),$$

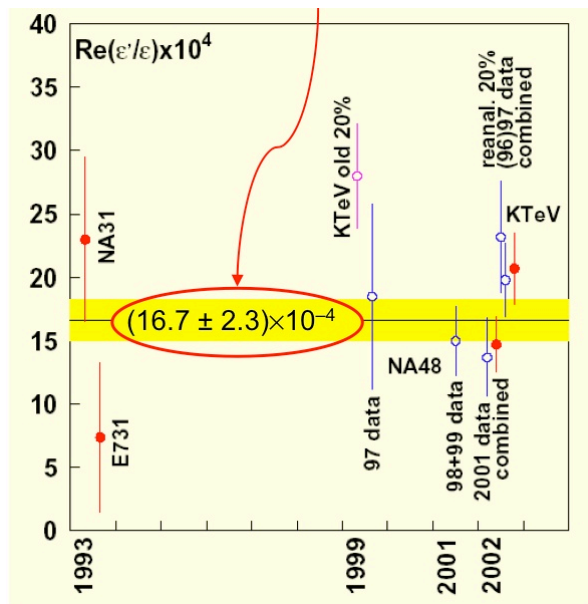
- Here again the weak interaction part is overwhelmed by theoretical hadronic uncertainties.
- Yet, it deserves some interest as the only kaon observables considered in the KM global fit.
- This CP-violating observable yields a complementary constraint to for instance the weak phase of the B mixing.





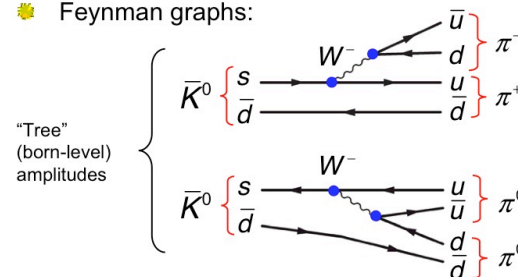
2.3 Aparté: direct CP violation in K decays.

- Not only the CP violation in the kaon mixing has been measured but also the direct CP violation in the kaon decay.
- Modify slightly the ε_K definition to account for the interference between penguin and tree decays to two pions.



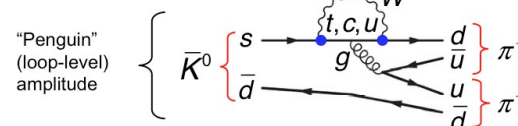
S. MONTPELLI

☀ Feynman graphs:



Interference

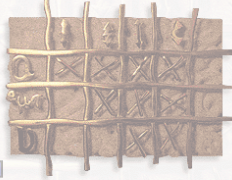
© A. Höcker.



- This is a very small effect and the first observation was reported in 2001 by NA48 and KTeV experiments, after 30 years of efforts.

- It happens that the SM prediction is plagued by hadronic uncertainties and makes unusable for the global fit this (in principle) very valuable information. Disregarded in the following.

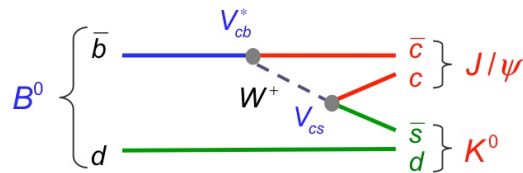
Lyon 2013



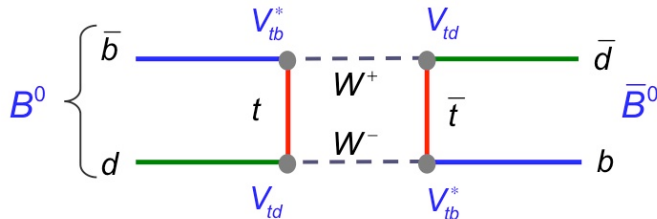
2.4 The measurement of $\sin(2\beta)$.

- Sketch of the method: double slit experiment.
- An interference between processes exhibiting V_{td} and V_{cb} matrix element:

- The $b \rightarrow c$ process: $V_{cb}^* V_{cs}$



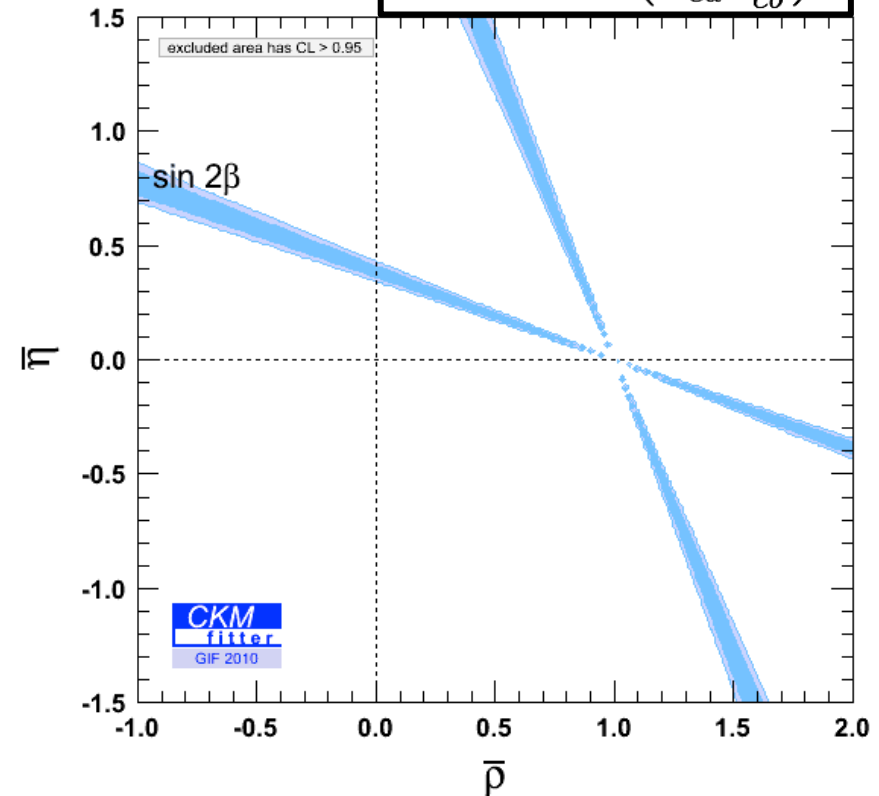
- The mixing process: $V_{tb}^* V_{td}$

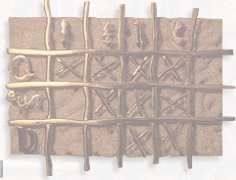


- And additionally the K^0 mix: $V_{cd} V_{cs}^*$:

$$\arg \left[\frac{A(B^0 \rightarrow J/\psi K^0)}{A(B^0_{\text{mix}} \rightarrow \bar{B}^0 \rightarrow J/\psi \bar{K}^0 \rightarrow J/\psi K^0)} \right] = \arg \left[\frac{V_{cs} V_{cb}^*}{(V_{td} V_{tb}^*)^2 \cdot V_{cs}^* V_{cb} \cdot (V_{cd} V_{cs})^2} \right] = \arg \left[\frac{(V_{cd} V_{cb}^*)^2}{(V_{td} V_{tb}^*)^2} \right] = -2\beta$$

$$\beta = \pi - \arg \left(\frac{V_{td} V_{tb}^*}{V_{cd} V_{cb}^*} \right),$$





2.4 The measurement of $\sin(2\beta)$.

- Sketch of the method: some definitions.

$$\beta = \pi - \arg \left(\frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} \right),$$

- The CP asymmetry:

$$A_{CP}(f, t) = \frac{N(\bar{B}^0(t) \rightarrow f) - N(B^0(t) \rightarrow f)}{N(\bar{B}^0(t) \rightarrow f) + N(B^0(t) \rightarrow f)}$$

- Can be expressed as a function of the S and C observables:

$$A_{CP}(f, t) = S \sin(\Delta m_{dt}) - C \cos(\Delta m_{dt})$$

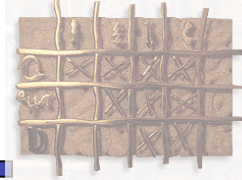
- which can be related to CP violating phase β :

$$\lambda = \frac{q}{p} \frac{A(\bar{B}^0 \rightarrow f)}{A(B^0 \rightarrow f)} = e^{-i2\beta} \frac{\bar{A}_f}{A_f}$$

- Let's notice that the charmless CP final state $\pi\pi$ would receive $S = \sin(2\alpha)$ in absence of penguin diagrams.

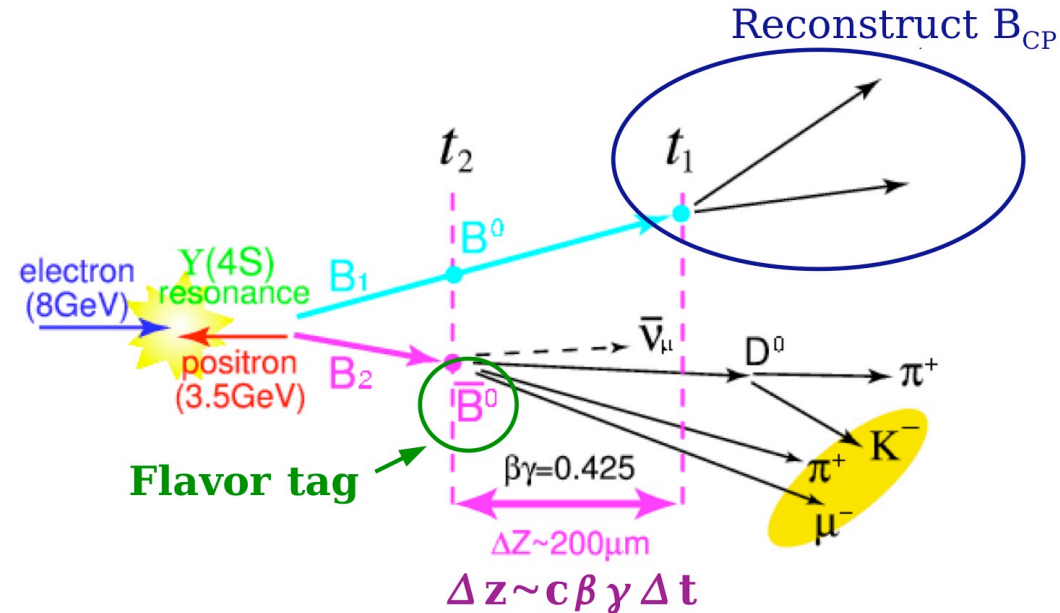
$$S = \frac{2\text{Im}\lambda}{1 + |\lambda|^2}$$

$$S = -\eta_{CP} \sin(2\beta)$$

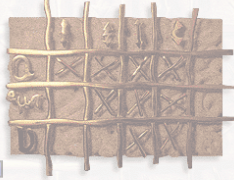


2.4 The measurement of $\sin(2\beta)$.

- The experimental method to measure S and C parameters:
- Fully reconstruct the bccs CP decay .
- Tag the flavour with the other B of the event.
- Reconstruct the time difference between the decays from the vertex separation.

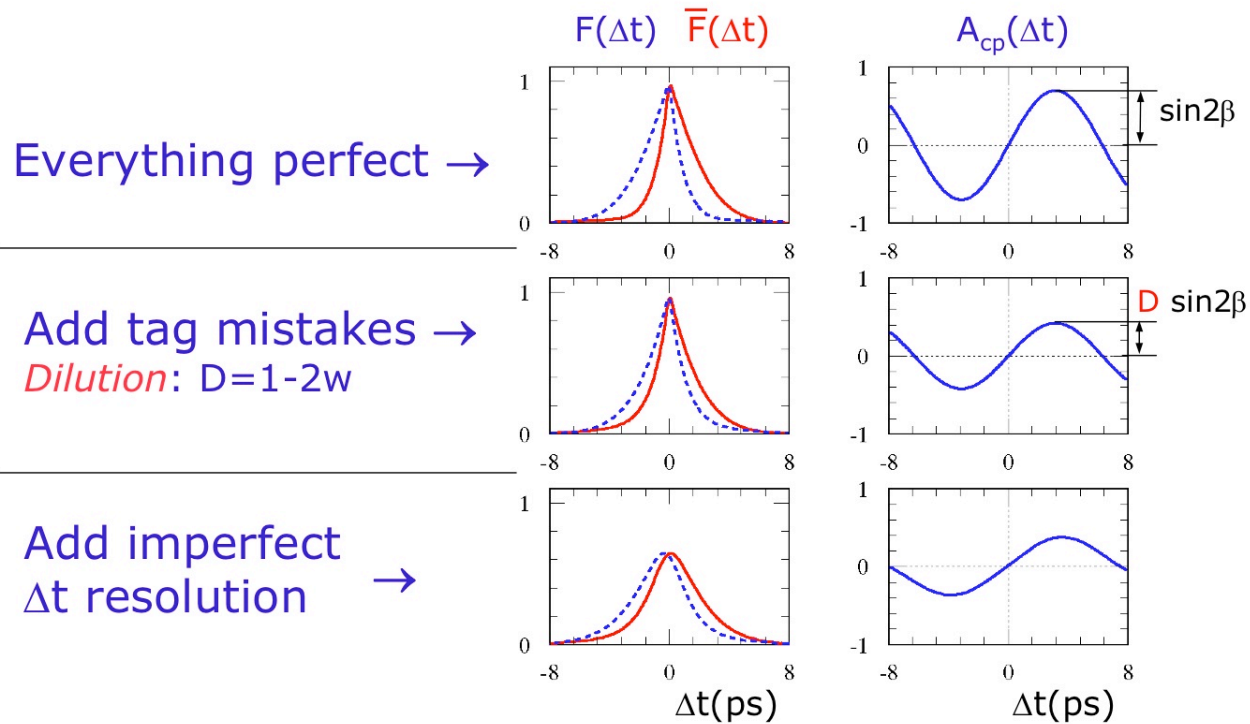


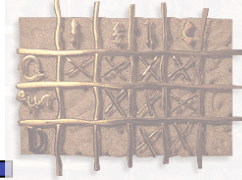
$$\frac{dP_{\text{sig}}}{dt}(\Delta t, \mathbf{q}) = \frac{e^{-|\Delta t|/\tau_B}}{4\tau_B} (1 + \mathbf{q}(\mathbf{S} \sin(\Delta m_d \Delta t) + \mathbf{A} \cos(\Delta m_d \Delta t)))$$



2.4 The measurement of $\sin(2\beta)$.

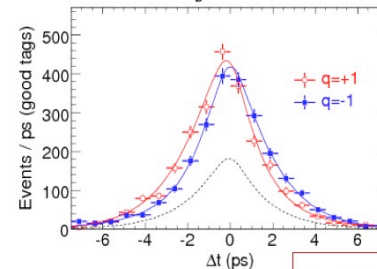
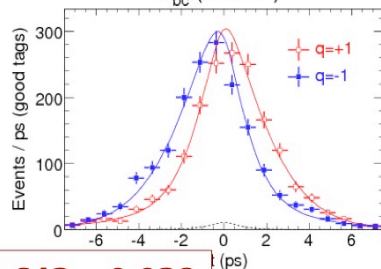
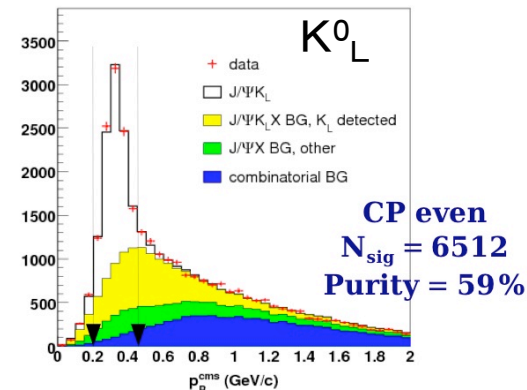
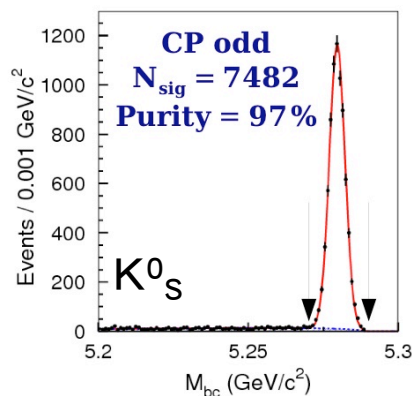
- Dilution factors: mistag rate and vertexing resolution.





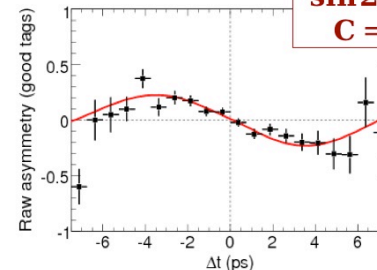
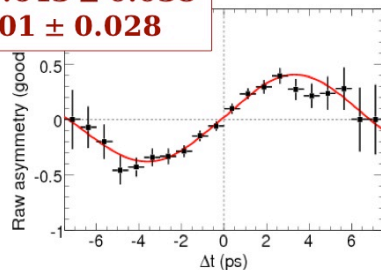
2.4 The measurement of $\sin(2\beta)$.

- A selection of Belle results as an illustration of this fantastic achievement.

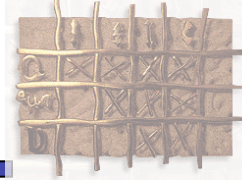


$\sin 2\beta = 0.643 \pm 0.038$
 $C = 0.001 \pm 0.028$

$\sin 2\beta = 0.641 \pm 0.057$
 $C = -0.045 \pm 0.033$

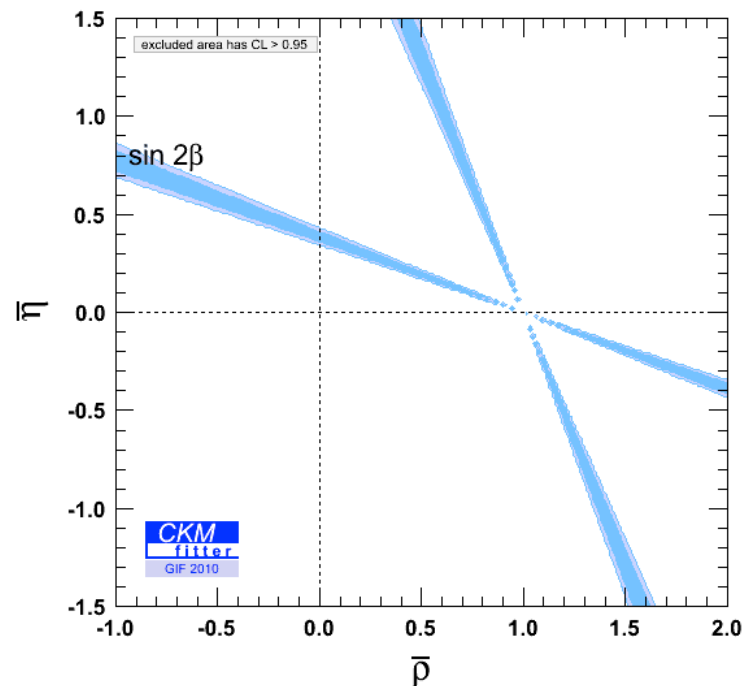
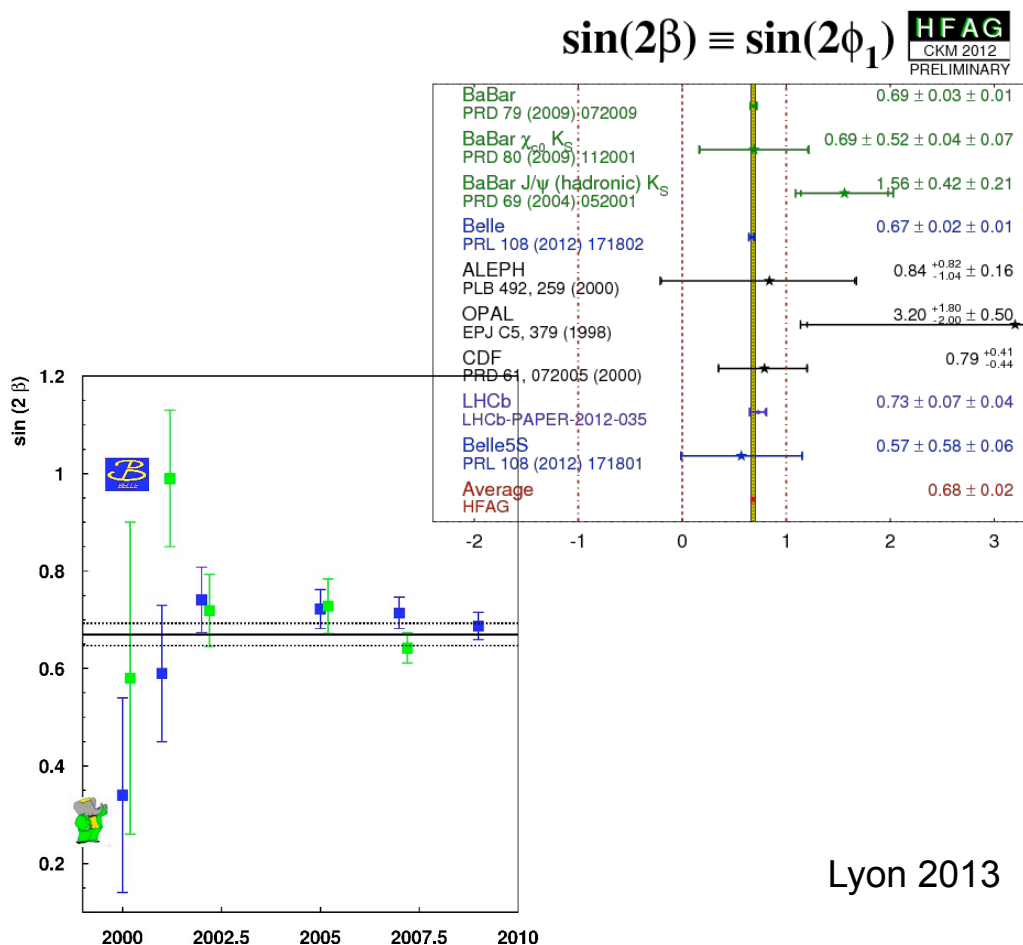


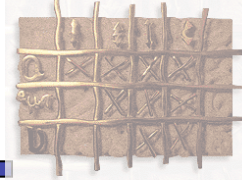
© K. Trabelsi.



2.4 The measurement of $\sin(2\beta)$.

- This measurement was the highlight of the physics case of the B factories and the accuracy of their measurements is a tremendous success...

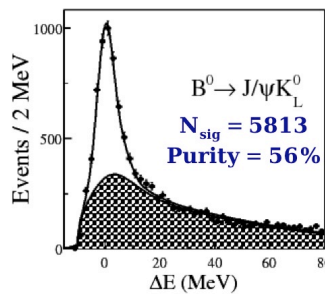
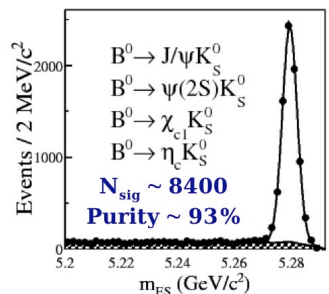




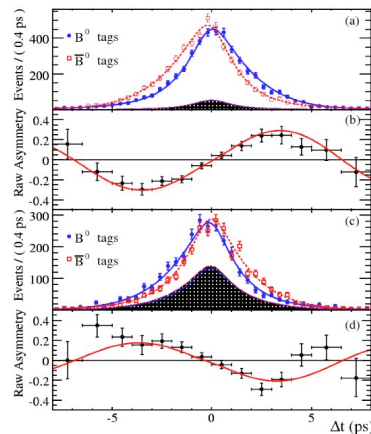
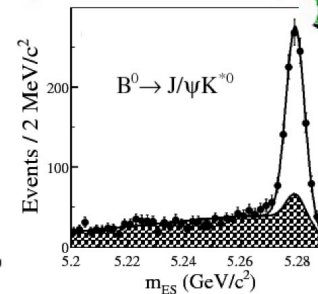
2.4 The measurement of $\sin(2\beta)$.

- Other charmonia modes are measured with good precision and nice consistency :

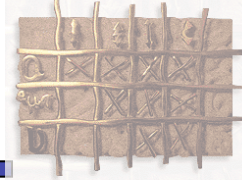
$\sin 2\beta$ in $(c\bar{c}) K^{(*)0}$



465×10^6 BB pairs
[ArXiv:0902.1708]



Mode	$\sin 2\beta$
$J/\psi K_S$	$0.657 \pm 0.036 \pm 0.012$
$J/\psi K_L$	$0.694 \pm 0.061 \pm 0.031$
$J/\psi K^0$	$0.666 \pm 0.031 \pm 0.013$
$\psi(2S) K_S$	$0.897 \pm 0.100 \pm 0.036$
$\chi_{c1} K_S$	$0.614 \pm 0.160 \pm 0.040$
$\eta_c K_S$	$0.925 \pm 0.160 \pm 0.057$
$J/\psi K^{*0}$	$0.601 \pm 0.239 \pm 0.087$
$c\bar{c} K^{(*)0}$	$0.687 \pm 0.028 \pm 0.012$

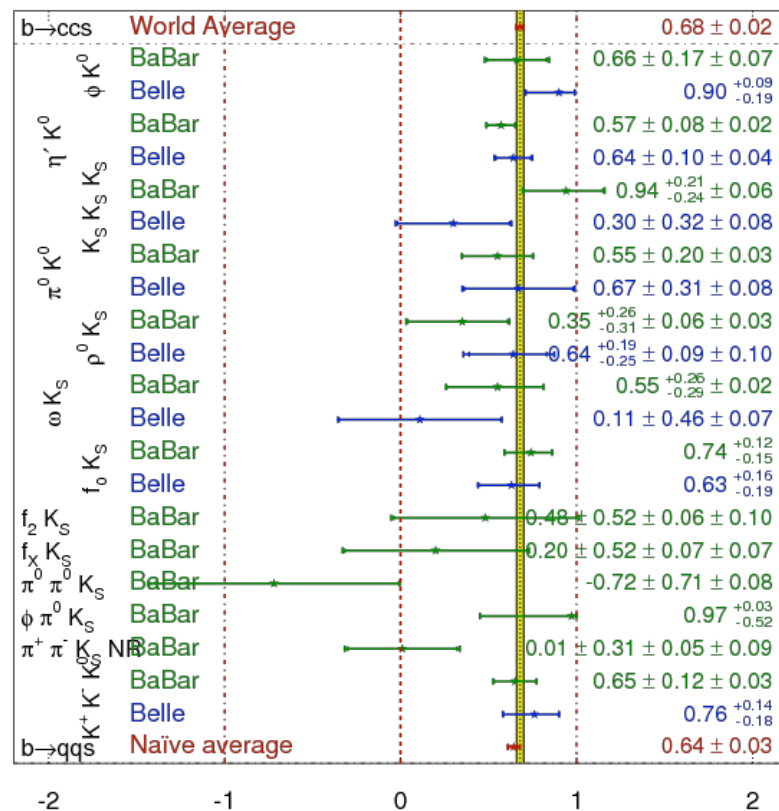


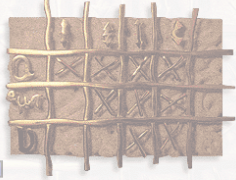
2.4 The measurement of $\sin(2\beta)$.

$$\sin(2\beta^{\text{eff}}) \equiv \sin(2\phi_1^{\text{eff}})$$

HFAG
Moriond 2012
PRELIMINARY

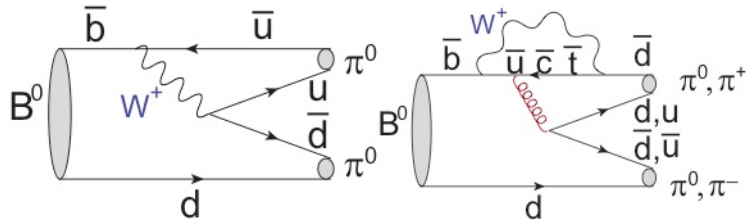
- An active search for β measurements concern the charmless decays proceeding through penguins ($b \rightarrow s, ss[qq]$).
- Probing a difference with $\sin(2\beta)$ measured in ($b \rightarrow ccs$) would be an indication of New Physics contributions in the loop diagrams (See Yasmine's seminar).
- The precision starts to be interesting but more statistics is crucial since each mode receives its own hadronic correction. The consistency is acceptable.





2.5 The angle α from $B \rightarrow \pi\pi$, $B \rightarrow \rho\rho$ (and $B \rightarrow \rho\pi$)

- The angle α can be analogously to β measured in the time dependent interference between the mixing and the decay of tree-mediated $b \rightarrow uud$ processes.
- The situation is further complicated by the presence of penguin diagram exhibiting a different CKM phase:



$$A(B^0 \rightarrow \pi^+ \pi^-) = T^{+-} e^{i\gamma} + P$$

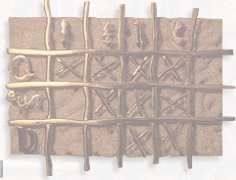
- The CP asymmetry is modified as:

$$\begin{aligned} A(t) &= S_{\pi^+ \pi^-} \sin(\Delta m t) - C_{\pi^+ \pi^-} \cos(\Delta m t) \\ &= \sqrt{1 - C_{\pi^+ \pi^-}^2} \sin 2\alpha_{\text{eff}} \sin(\Delta m t) - C_{\pi^+ \pi^-} \cos(\Delta m t) \end{aligned}$$

$$S_{\pi^+ \pi^-} = \sin 2\alpha + 2r \cos \delta \sin(\beta + \alpha) \cos 2\alpha + O(r^2)$$

$$r = |P|/|T|$$

- Additional information is required if you want to make the electroweak interpretation of the measurement.



2.5 The angle α from $B \rightarrow \pi\pi$

- Use companion modes $(\pi\pi)^{+/-0}$ and isospin symmetry to disentangle penguin contributions:

- completely general isospin decomposition

Gronau, London (1990)

$$A_{+-} = \langle \pi^+ \pi^- | H | B^0 \rangle = -A_{1/2} + \frac{1}{\sqrt{2}} A_{3/2} - \frac{1}{\sqrt{2}} A_{5/2}$$

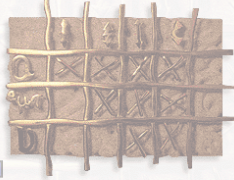
$$A_{00} = \langle \pi^0 \pi^0 | H | B^0 \rangle = \frac{1}{\sqrt{2}} A_{1/2} + A_{3/2} - A_{5/2}$$

$$A_{+0} = \langle \pi^+ \pi^0 | H | B^+ \rangle = \frac{3}{2} A_{3/2} + A_{5/2}$$

- Tree and EWP contribute to $|\Delta I|=1/2$ and $3/2$ amplitudes
- QCD penguins contribute to $|\Delta I|=1/2$ amplitudes
- $|\Delta I|=5/2$ induced by Isospin Symmetry Breaking (not present in H_W)
- Neglecting $|\Delta I|=5/2$ transition and EWP, A_{+0} is pure Tree.
- Isospin triangular relation :

$$A_{+-} + \sqrt{2} A_{00} = \sqrt{2} A_{+0}$$

$$\bar{A}_{+-} + \sqrt{2} \bar{A}_{00} = \sqrt{2} \bar{A}_{+0}$$

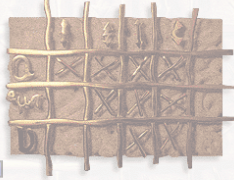


2.5 The angle α from $B \rightarrow \pi\pi$

- Use companion modes ($\pi\pi$) and isospin symmetry to disentangle penguin contributions.
- In addition to the time-dependent analysis parameters S and C , consider the Branching Fractions of the companion modes.
- Geometrical resolution:

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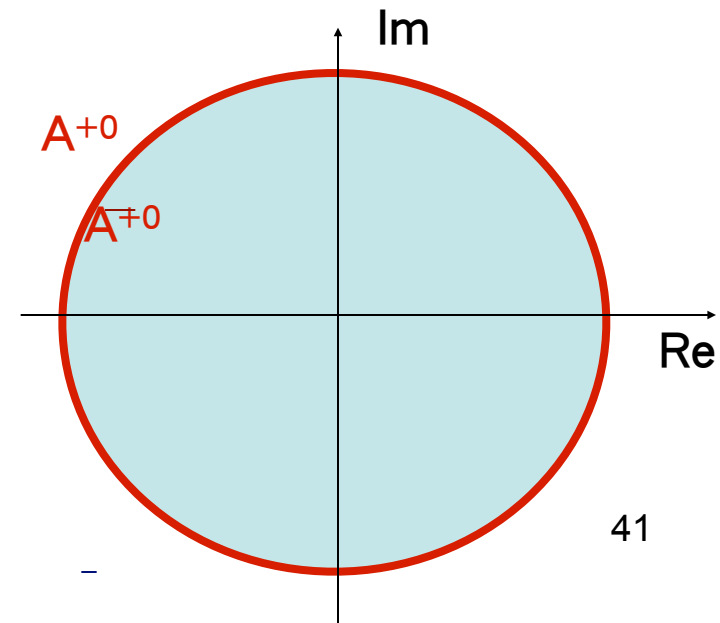


2.5 The angle α from $B \rightarrow \pi\pi$

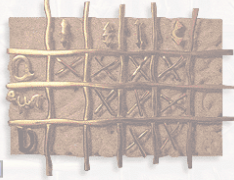
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- Geometrical resolution:

• $B^{+0} \rightarrow |A^{+0}| = |A^{\mp 0}|$

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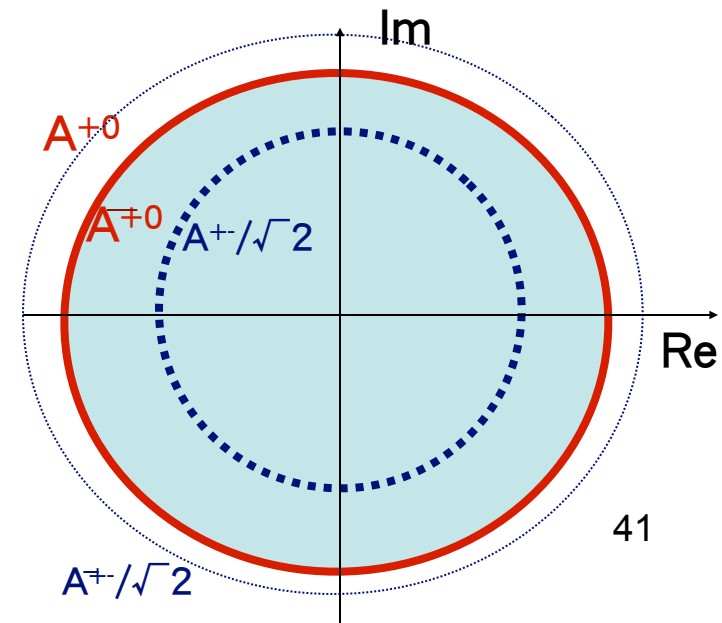


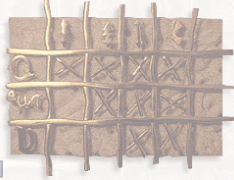
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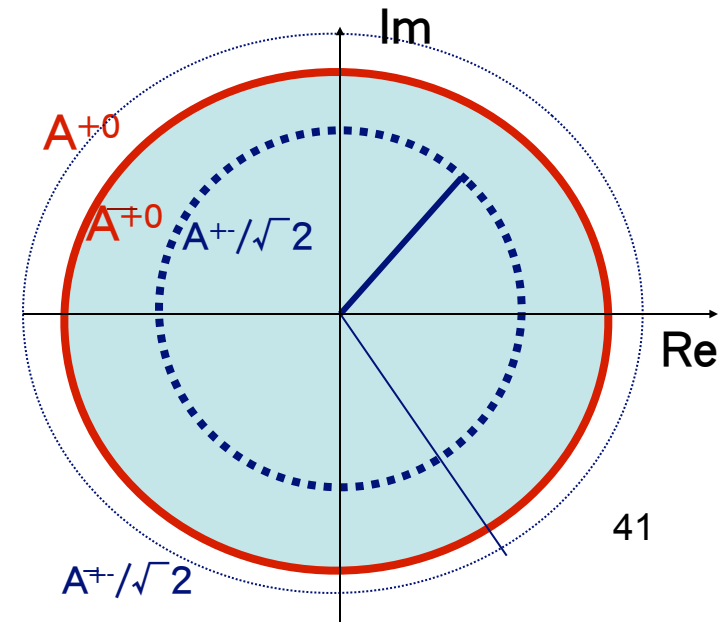
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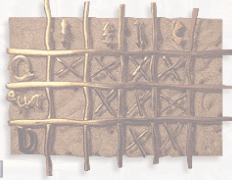
• Geometrical resolution:

- $B^{+0} \rightarrow |A^{+0}| = |A^{\mp 0}|$
- $B^{+-}, C^{+-} \rightarrow |A^{++}|, |A^{+-}|$
- $S^{+-} \rightarrow \sin(2\alpha_{\text{eff}}) \rightarrow 2\text{-fold } \alpha_{\text{eff}} \text{ in } [0, \pi]$

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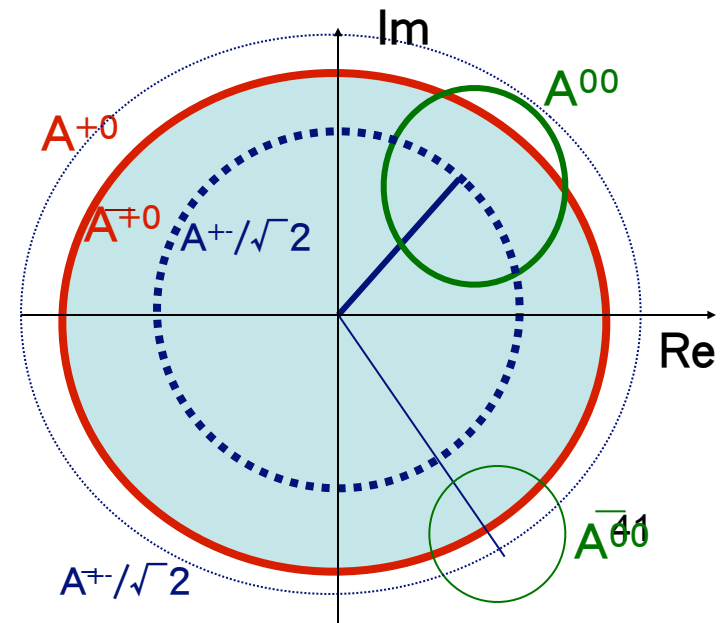
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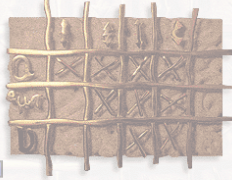
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- B^{00}, C^{00} $\rightarrow |A^{00}|, |A^{\bar{0}0}|$

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2.5 The angle α from $B \rightarrow \pi\pi$

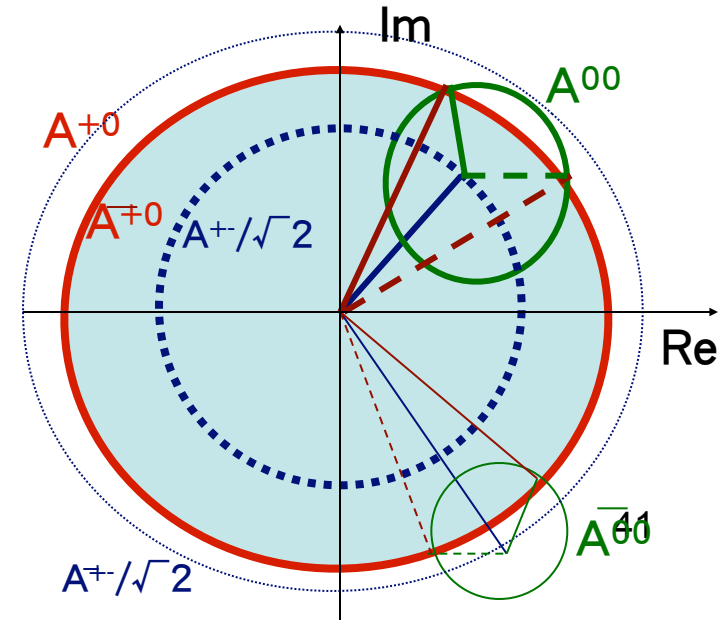
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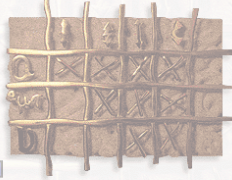
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- B^{00}, C^{00} $\rightarrow |A^{00}|, |A^{\bar{0}0}|$

• Closing SU(2) triangle $\rightarrow 8\text{-fold } \alpha$

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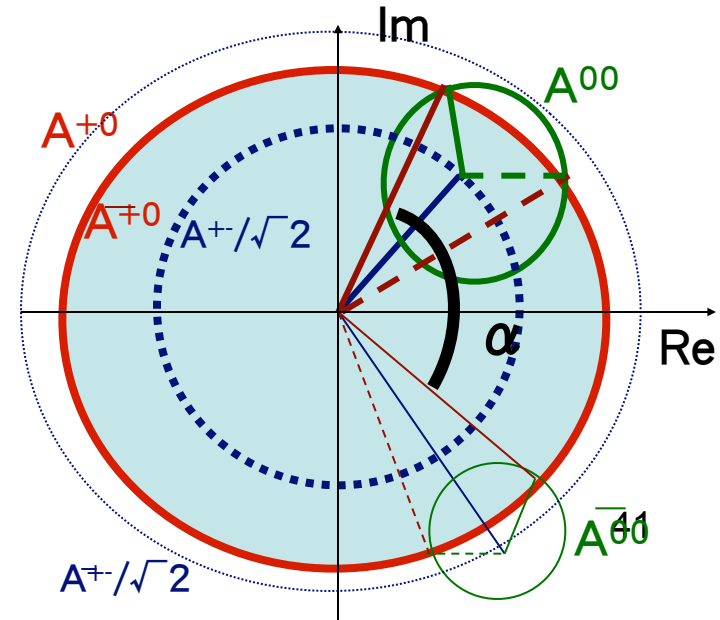
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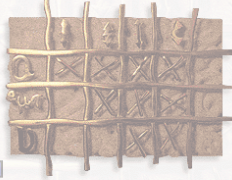
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- B^{00}, C^{00} $\rightarrow |A^{00}|, |A^{\bar{0}0}|$
- Closing SU(2) triangle $\rightarrow 8\text{-fold } \alpha$
- S^{00} \rightarrow relative phase between A^{00} & $A^{\bar{0}0}$

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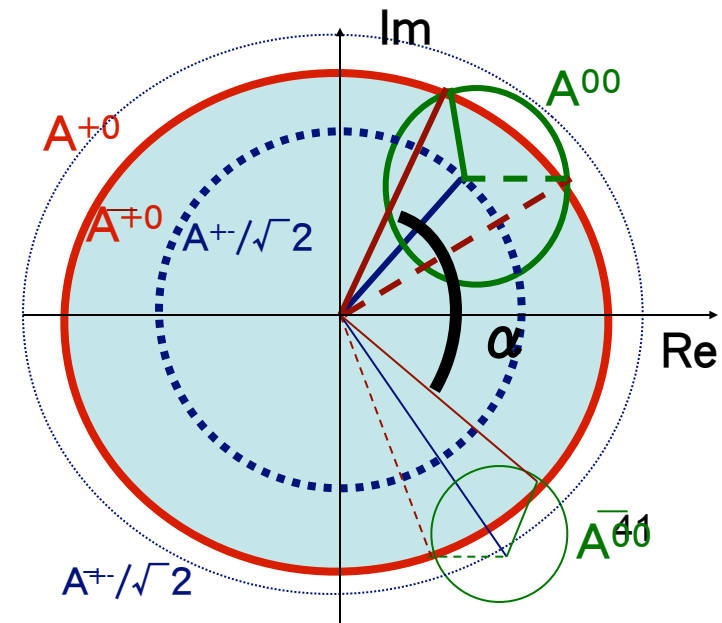
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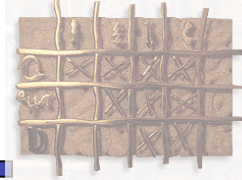
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- B^{00}, C^{00} $\rightarrow |A^{00}|, |A^{\bar{0}0}|$
- Closing SU(2) triangle $\rightarrow 8\text{-fold } \alpha$
- S^{00} \rightarrow relative phase between A^{00} & $A^{\bar{0}0}$

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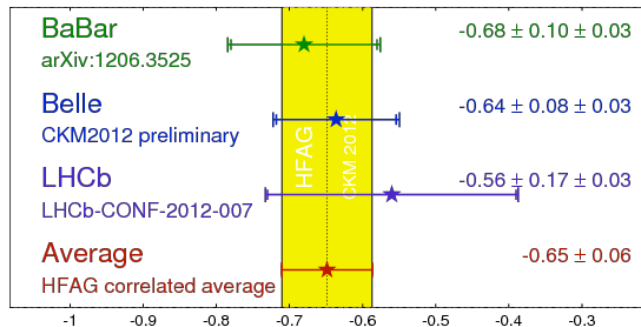




2.5 The angle α from $B \rightarrow \pi\pi$

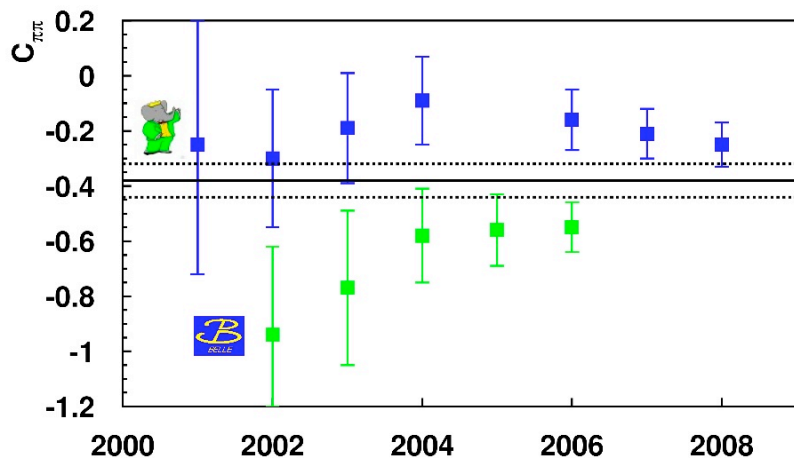
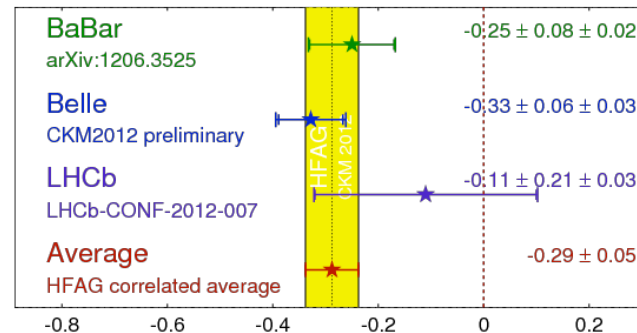
$\pi^+ \pi^- S_{CP}$

HFAG
CKM 2012
PRELIMINARY



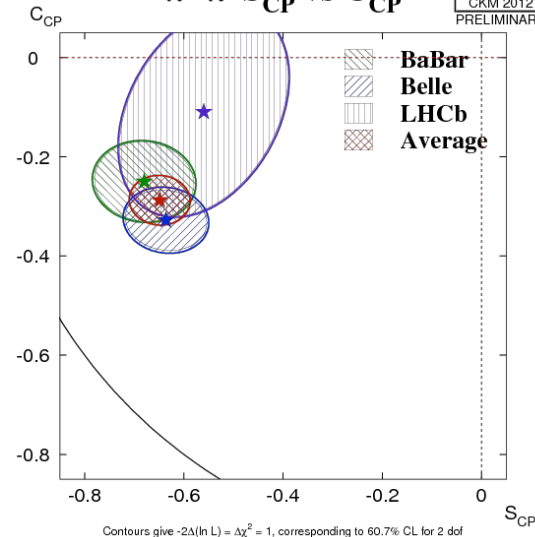
$\pi^+ \pi^- C_{CP}$

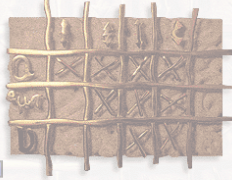
HFAG
CKM 2012
PRELIMINARY



$\pi^+ \pi^- S_{CP}$ vs C_{CP}

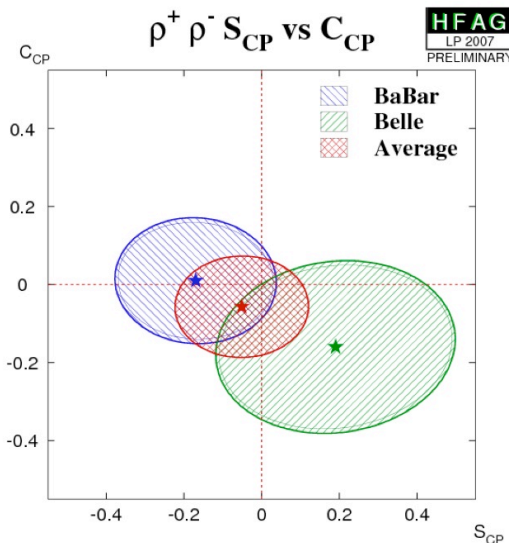
HFAG
CKM 2012
PRELIMINARY



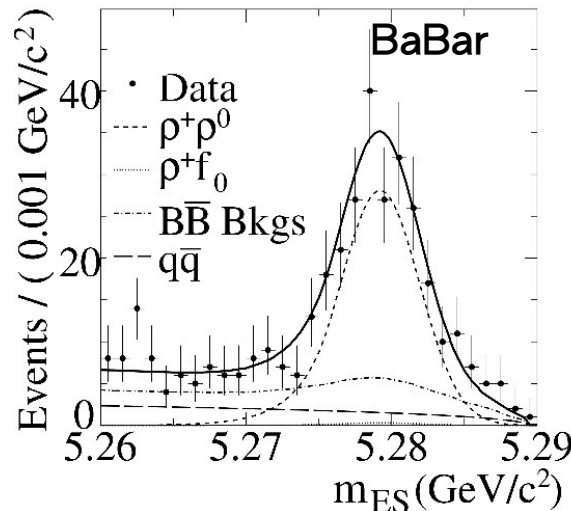


2.5 The angle α from $B \rightarrow \rho\rho$

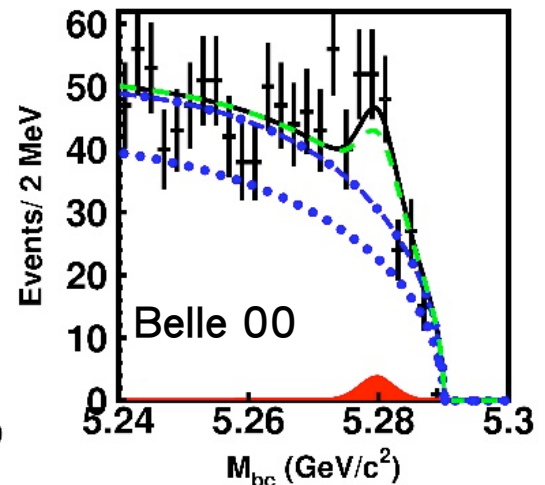
- $B \rightarrow VV$ process but final state almost pure CP state as the decay is saturated with longitudinally polarized ρ 's.
- Isospin decomposition same as $B \rightarrow \pi\pi$ at first order.
- The inputs of the analysis: $(\text{Br}(B \rightarrow \rho^+ \rho^-), S_{\rho^+ \rho^-}, C_{\rho^+ \rho^-}, \text{Br}(B \rightarrow \rho^+ \rho^0), \text{Br}(B \rightarrow \rho^0 \rho^0)) + f_L$

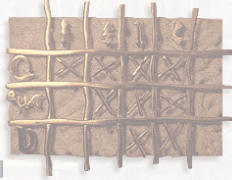


S.Monteil



Lyon 2013

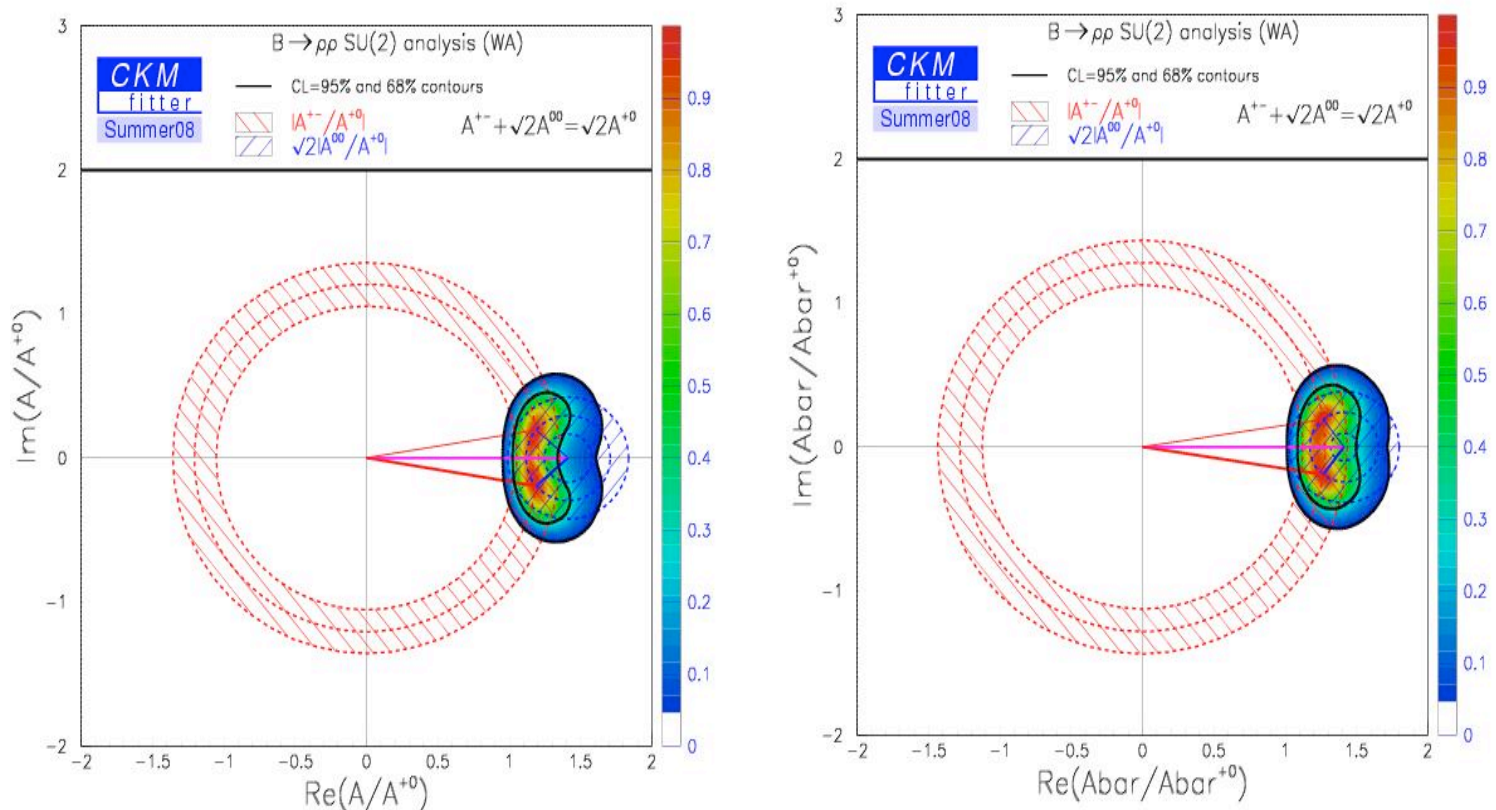




2.5 The angle α from $B \rightarrow \rho\rho$

Inputs :

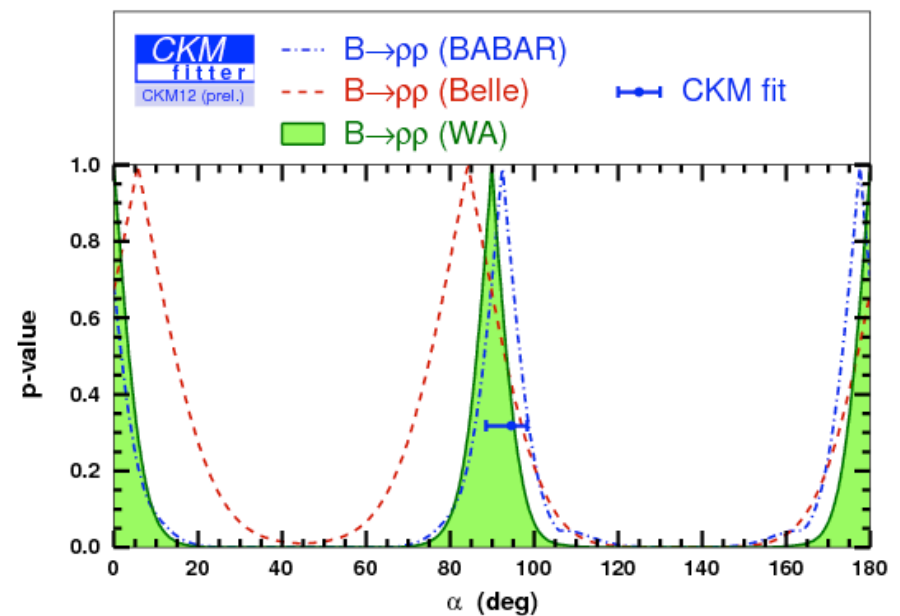
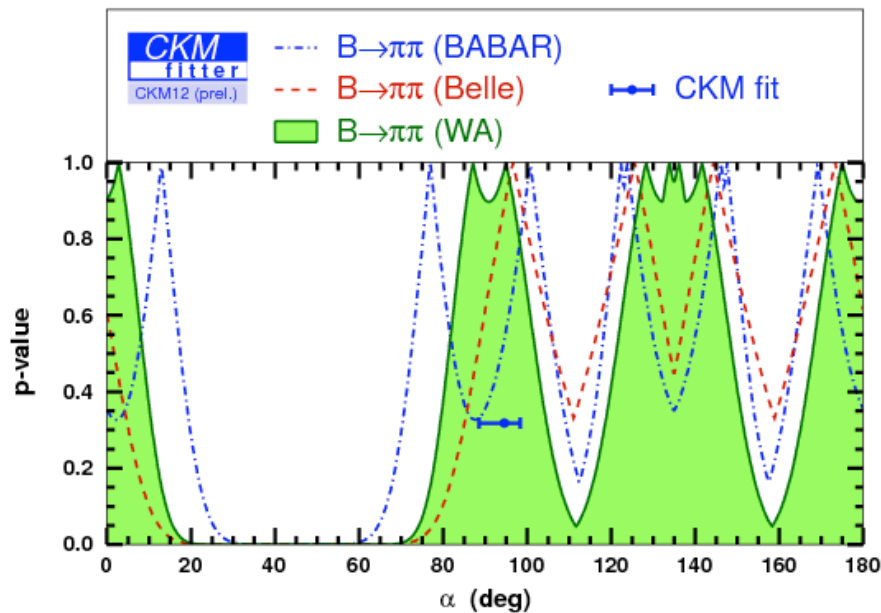
B^{+-}
 B^{0+}
 B^{00}
 C^{+-}
 C^{00}
 S^{00}
 f_L^{+-}
 f_L^{0+}
 f_L^{00}



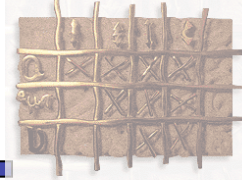
Both triangles (squashed because of the smallness of B^{00}) do close \rightarrow 8-fold solution for alpha but S^{00} breaks the degeneracy.



2.5 The angle α : World Averages

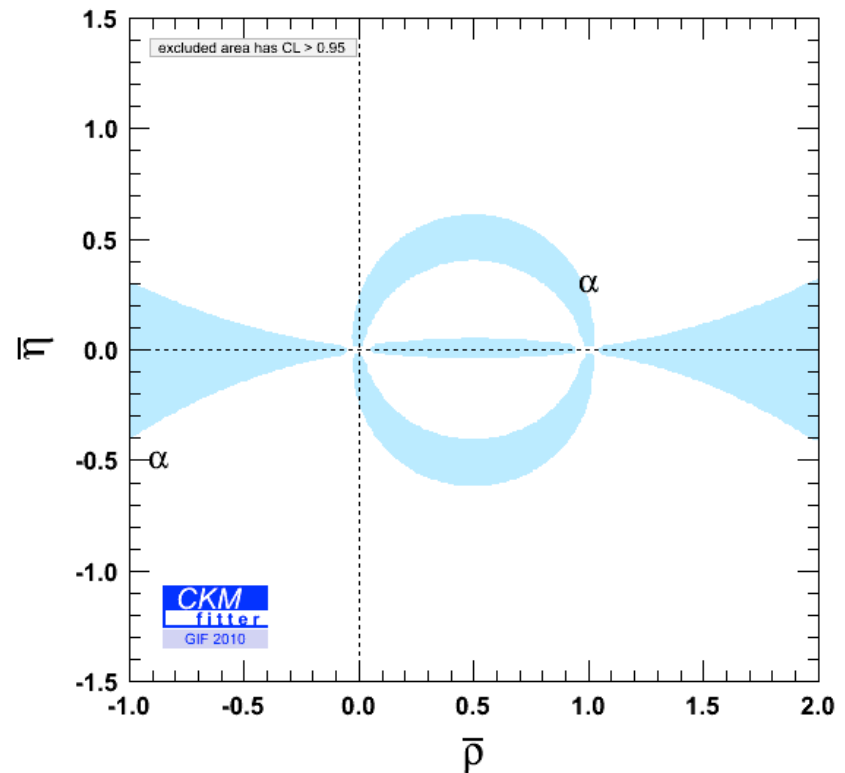
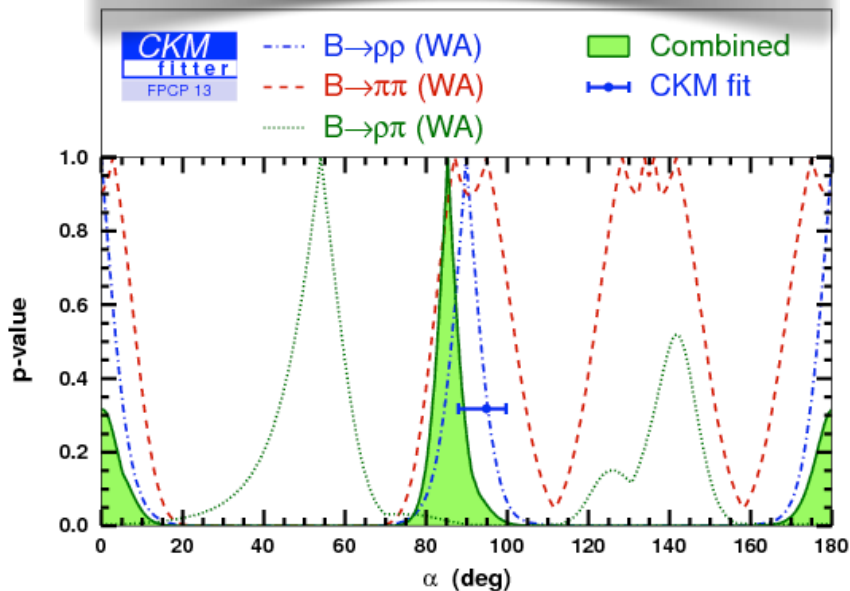


Nice consistency between BaBar and Belle measurements, as well as between $B \rightarrow \rho\rho$ and $B \rightarrow \pi\pi$.

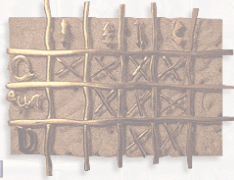


2.5 The angle α : WA

$$\alpha(\text{WA}) = (85.4^{+4.0}_{-3.8})^\circ.$$

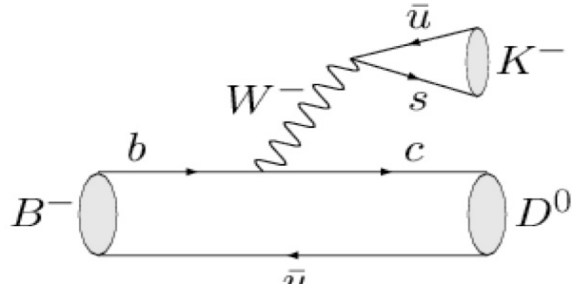


$B \rightarrow \rho\pi$ dominates the average. 5% (!) precision measurements.



2.6 The angle γ : principle of the measurement

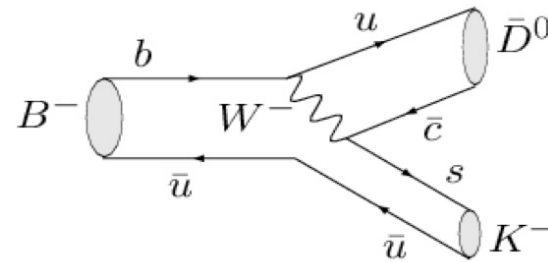
- The determination of the angle γ requires interferences between charmless $b \rightarrow u$ transition and another weak phase, say for instance $b \rightarrow c$. This interference is realized in decays $B \rightarrow DK$.



$$A(B^- \rightarrow D^0 K^-) = a$$

$\downarrow CP$

$$\bar{A}(B^+ \rightarrow \bar{D}^0 K^+) = a$$



$$A(B^- \rightarrow \bar{D}^0 K^-) = a e^{-i\gamma} r_B e^{i\delta_B}$$

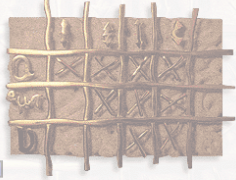
$\downarrow CP$

$$\bar{A}(B^+ \rightarrow D^0 K^+) = a e^{+i\gamma} r_B e^{i\delta_B}$$

- The interference level between $b \rightarrow u$ and $b \rightarrow c$ transitions is controlled by the parameter r_B :

$$r_B = \left| \frac{A(B^- \rightarrow \bar{D}^0 K^-)}{A(B^- \rightarrow D^0 K^-)} \right|$$

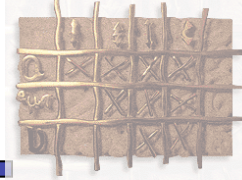
- No penguin: theoretically clean. But one has to reach through undistinguishable paths the same final state !



2.6 The angle γ : the methods

- We hence have to reconstruct the D mesons in final states accessible to both D^0 and $anti-D^0$. There are three main techniques which have been undertaken at B factories:
1. GLW (Gronau, London, Wyler): search for D mesons decays into 2-body CP eigenstates, e.g K^+K^- , $\pi^+\pi^-$ (CP=+) or $K_S\pi^0$, ϕK_S (CP=-). Somehow natural but very low branching fractions.
 2. ADS (Atwood, Dunietz, Soni): Use $anti-D^0 \rightarrow K^-\pi^+$ for $b \rightarrow u$ transitions (Cabibbo allowed) and $D^0 \rightarrow K^-\pi^+$ (Doubly Cabibbo suppressed) for $b \rightarrow c$ transitions. Again low branching fractions and additionally one has to know the strong phase of the D decay.
 3. GGSZ (Giri, Grossman, Sofer, Zupan): use quasi 2-body CP eigenstates of the D to be resolved in the Dalitz plane. $D \rightarrow K_S\pi^+\pi^-$. So far the most precise gamma determination.

Note: I used D^0K for illustration. The same stands for D^*K and DK^* . The hadronic factors (r_B, δ_B) are however different in each case.



2.6 The angle γ : a closer look to GGSZ

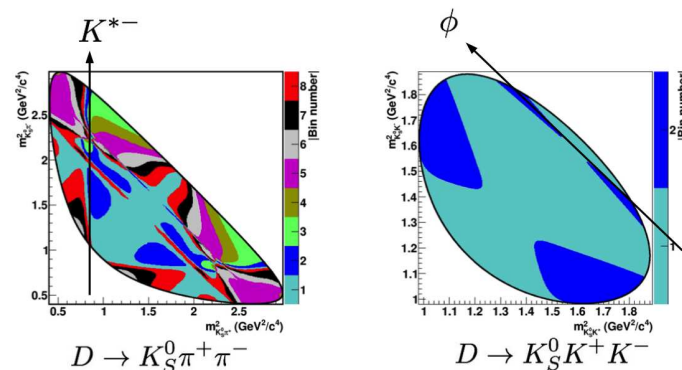
- The comparison of the Dalitz planes (DP) of the decays $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ or $K_S^0 K^+ K^-$ for the transitions $B^+ \rightarrow DK^+$ and $B^- \rightarrow DK^-$ contains information on γ angle.
- Constrain from CLEO-c measurements the strong phase variation in DP. (Phys. Rev. D 82 (2010) 112006)

- DP binned in regions of similar strong phase:

- Defining:

$$x_{\pm} = r_B \cos(\delta_B \pm \gamma),$$

$$y_{\pm} = r_B \sin(\delta_B \pm \gamma).$$

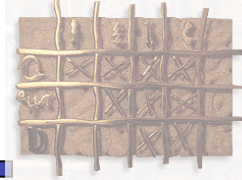


- One counts the number of events in each bins i for B^+ and B^- :

$$N_{\pm i}^+ \propto K_{\mp i} + (x_+^2 + y_+^2)K_{\pm i} + 2\sqrt{K_i K_{-i}}[x_+ \cos \delta_D(\pm i) \mp y_+ \sin \delta_D(\pm i)],$$

$$N_{\pm i}^- \propto K_{\pm i} + (x_-^2 + y_-^2)K_{\mp i} + 2\sqrt{K_i K_{-i}}[x_- \cos \delta_D(\pm i) \mp y_- \sin \delta_D(\pm i)].$$

- And solve for the four unknowns x and y .



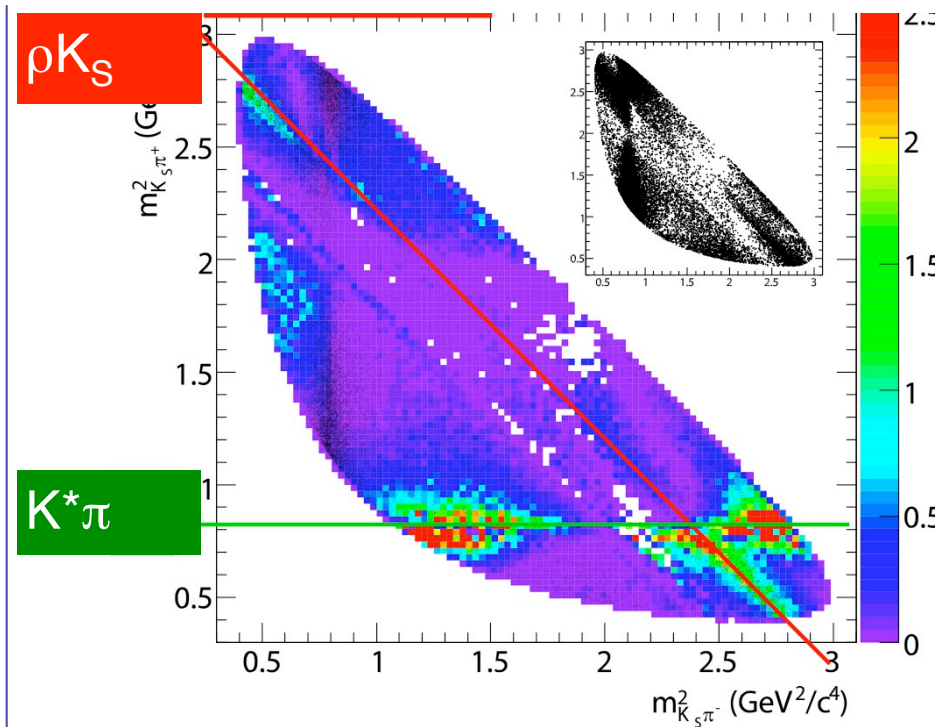
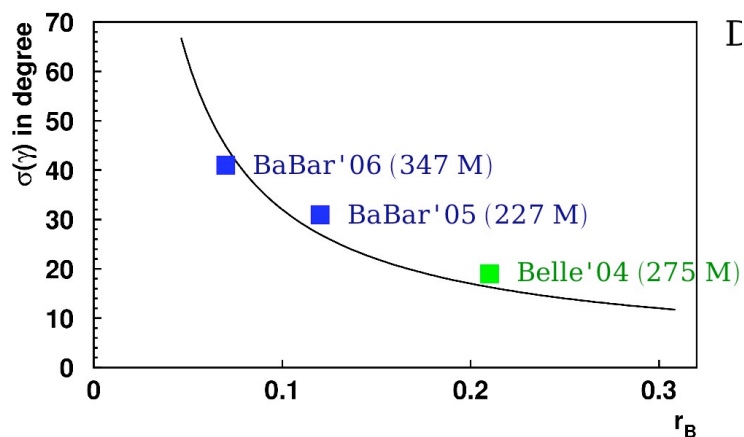
2.6 The angle γ : sensitivity GGSZ

- In the Dalitz plane, the level of interference is controlled by the cartesian coordinates (they are the experimental inputs for gamma extraction):

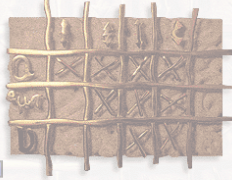
$$x_{\pm} = r_B \cos(\delta_B \pm \gamma)$$

$$y_{\pm} = r_B \sin(\delta_B \pm \gamma)$$

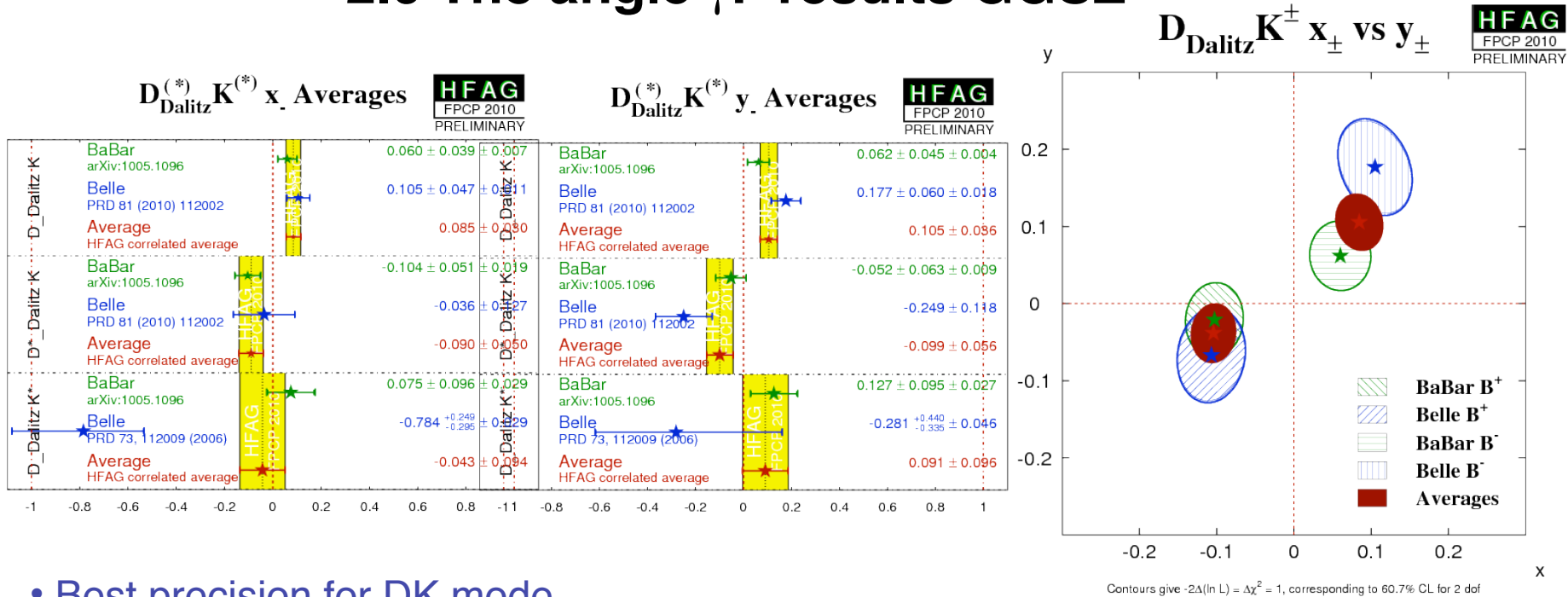
- How does the uncertainty on γ scales with r_B ? $1/r_B$...



- Sensitivity plot. Which regions of the Dalitz plane do contribute the more to the gamma precision: $K^*\pi$ and ρK_S bands.



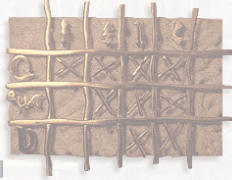
2.6 The angle γ : results GGSZ



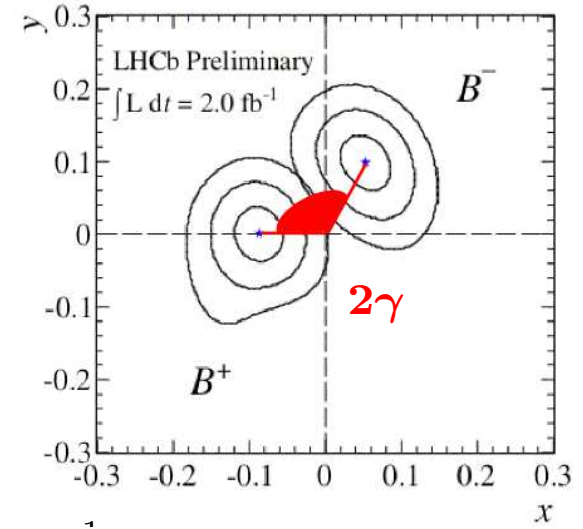
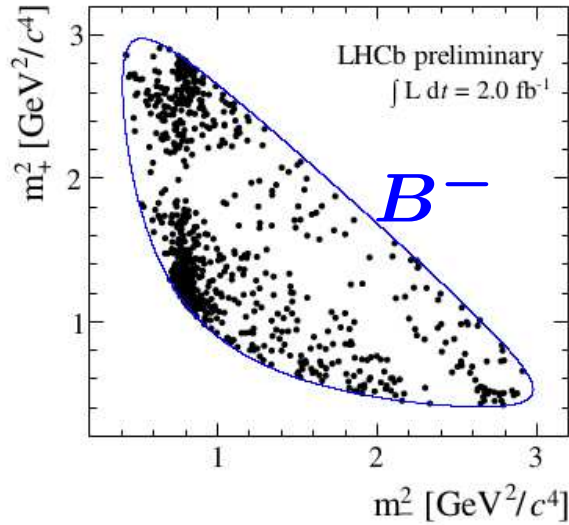
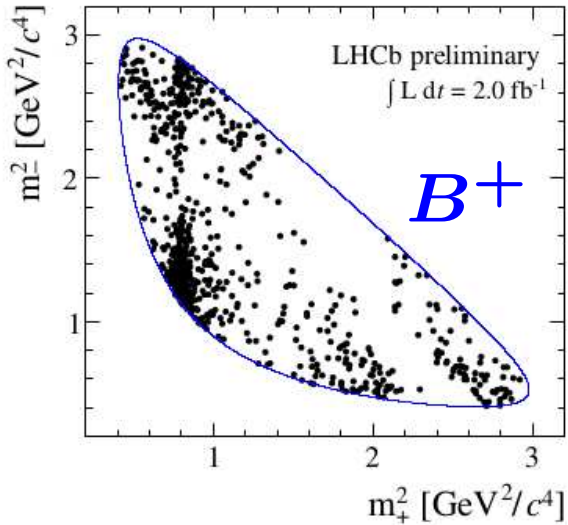
- Best precision for DK mode.
- The BaBar and Belle experiments extracted gamma using a frequentist scheme:

$$\gamma_{BaBar} = 69_{-14}^{+15} \text{ (stat.)} \pm 4 \text{ (syst.)} \pm 3 \text{ (mod.) deg}$$

$$\gamma_{Belle} = 78_{-12}^{+11} \text{ (stat.)} \pm 4 \text{ (syst.)} \pm 9 \text{ (mod.) deg}$$



2.6 The angle γ : results GGSZ LHCb

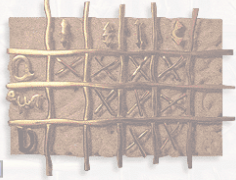


$$r_B = (8.8_{-2.4}^{+2.3})10^{-2},$$

$$\gamma = (57 \pm 16)^\circ$$

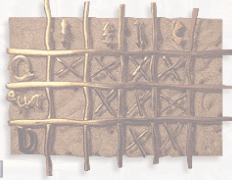
(2+1 /fb)

- Best precision for *DK* mode.

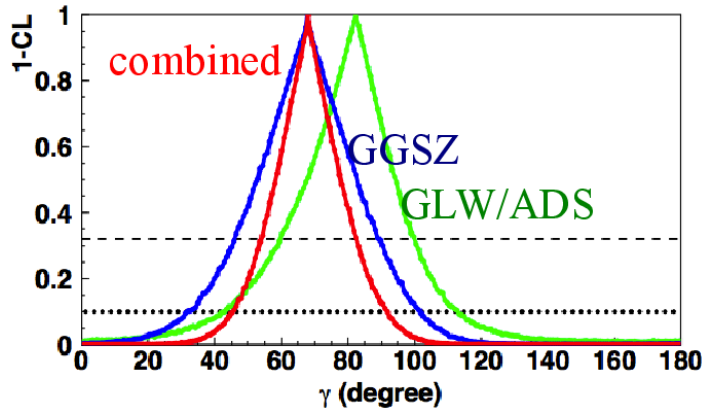


2.6 The angle γ : WA

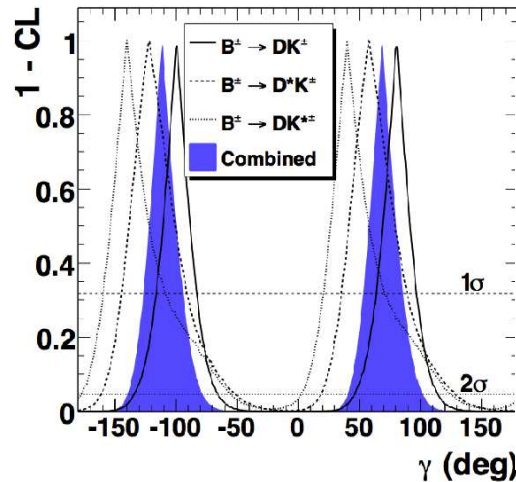
- GGSZ method is nowadays the best way to extract the γ angle. Other methods provide very valuable constraints on r_B and hence contribute to the overall precision.
- The high statistics expected at the LHC will allow to measure the γ angle with a precision comparable to what is achieved with α . LHCb already superseded B-factories precision w/ most of its results obtained w/ 1/fb.
- Though the less well determined angle of the Unitarity triangle, the γ angle measurement is a tremendous achievement of the B factories: was not fully foreseen at the beginning of their operation.
- The γ angle determination makes use of frequentist treatment (MC based) to ensure all the possible values of nuisance parameters (r_B in particular) are tested in the evaluation of the coverage. Significant variation on the global uncertainty w.r.t less sophisticated method. Mandatory for the time being.



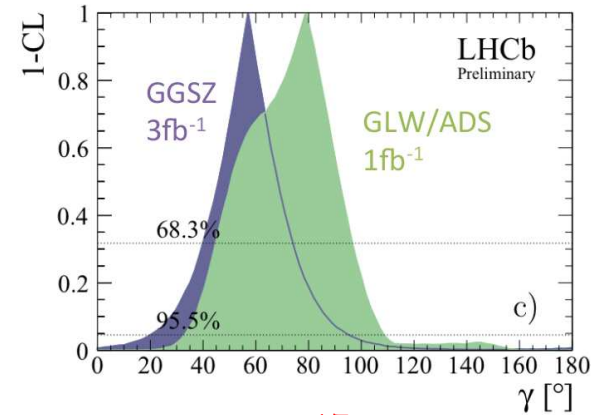
2.6 The angle γ : WA



arXiv:1301.2033

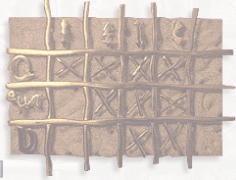


Phys Rev D 87, 052015 (2013)



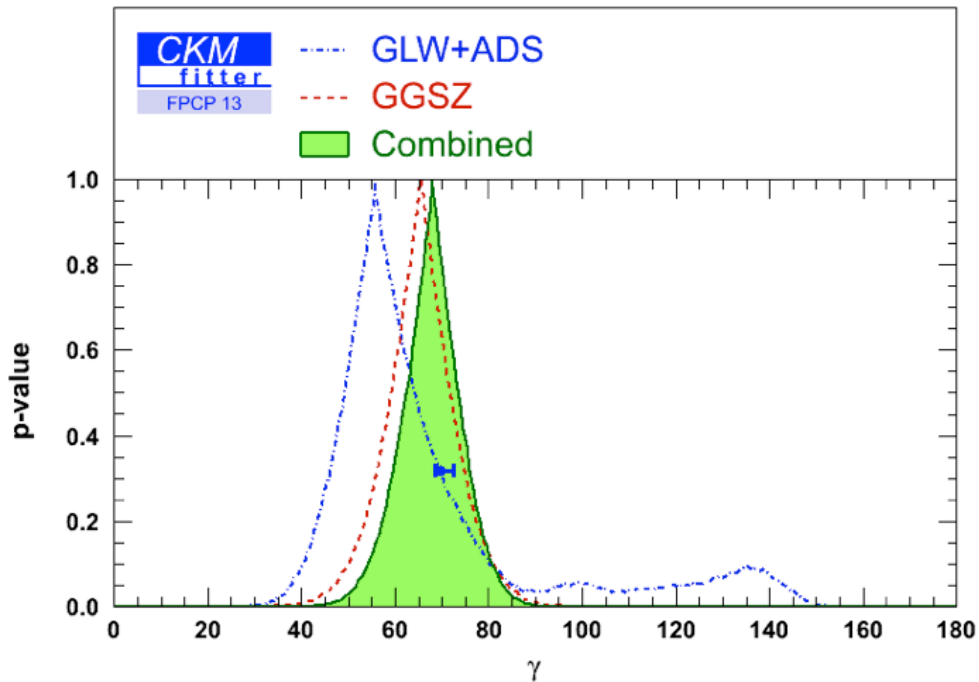
LHCb-CONF-2013-006

$$\begin{aligned} \gamma(\text{Belle}) &= (68^{+15}_{-14})^{\circ} \\ \gamma(\text{BaBar}) &= (69^{+17}_{-16})^{\circ} \\ \gamma(\text{LHCb}) &= (67 \pm 12)^{\circ} \end{aligned}$$

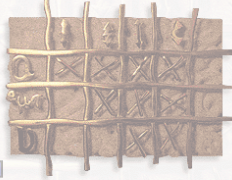


2.6 The angle γ : WA

- The γ angle grand combination BaBar/Belle/LHCb:

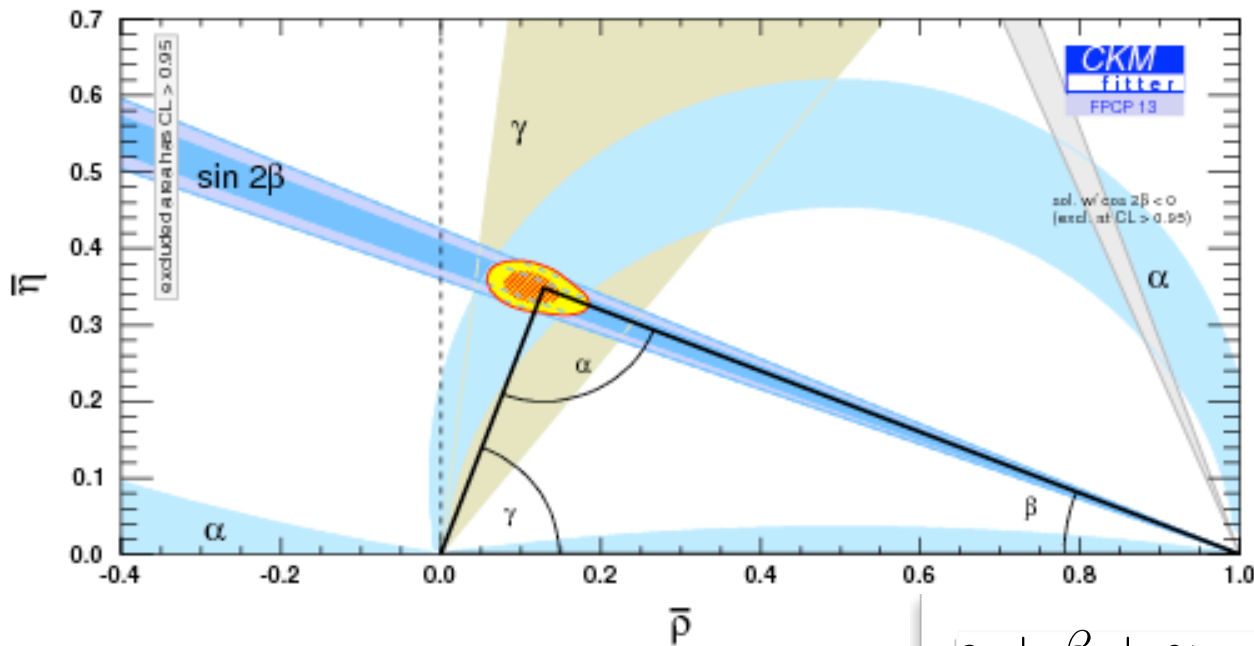


$$\gamma(\text{WA}) = (68.0^{+8.0}_{-8.5})^\circ.$$



2.7 Conclusion of Chapter 2 and introduction to Chapter 3

- We have now all the relevant experimental ingredients to produce the consistency check of all observables in the Standard Model and hence test the KM mechanism. By anticipation, we can produce a unitarity triangle with angles only:



$$\alpha + \beta + \gamma = (174.8 \pm 9.4)^\circ$$